

EXAMPLES OF COMPACT LOCALLY AFFINE SPACES

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ABSTRACT. A construction is given which exhibits many finitely-generated torsion-free nilpotent groups as fundamental groups of compact complete locally affine spaces.

1. Introduction. It is known (see [1] and [2], for example) that there is a close relationship between homogeneous spaces of solvable Lie groups (solvmanifolds) and complete locally affine spaces. One question arising in this connection is what discrete subgroups of solvable (in particular, nilpotent) Lie groups occur as fundamental groups of compact complete locally affine spaces. Of course, finitely-generated torsion-free abelian groups are such fundamental groups. So are finitely-generated torsion-free two-step nilpotent groups. Aside from this and examples of low dimension, not much seems to be known about fundamental groups of locally affine spaces.

In §2, we give a construction which exhibits a wide variety of finitely-generated torsion-free nilpotent groups (all of which are fundamental groups of compact nilmanifolds—see [5]) as fundamental groups of compact complete locally affine spaces. The construction suggests a problem about Lie algebras which we mention in §3.

2. Construction of examples. Let N be a nilpotent real Lie algebra possessing a nonsingular derivation D . Choose a basis $\{x_1, \dots, x_n\}$ of N and identify N with \mathbb{R}^n via

$$x_i \mapsto \begin{pmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$$

where the 1 is in the i th row. Then the map

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