BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 4, July 1971

GENERALIZED SOLUTIONS OF QUASILINEAR, DIFFERENTIAL INEQUALITIES. I. ELLIPTIC OPERATORS

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Communicated by Philip Hartman, December 28, 1970

Introduction. The purpose of this note, together with its sequel on parabolic inequalities, is to present various results describing the local and global behaviour of generalized solutions, subsolutions and supersolutions associated with quasilinear, second order differential operators of the forms

$$Qu = \operatorname{div} \mathcal{A}(x, u, Du) + \mathcal{B}(x, u, Du), \tag{1}$$

$$Pu = \operatorname{div} \mathcal{A}(x, t, u, Du) + \mathcal{B}(x, t, u, Du) - D_t u. \tag{2}$$

Here $x = (x_1, \cdots, x_n)$ represents a point in a Euclidean n-space E^n; t denotes a time variable; Du is the spatial gradient of the strongly differentiable function u; $\mathcal{A}$ and $\mathcal{B}$ are respectively measurable, n-vector and scalar functions of their arguments and div denotes the spatial divergence.

The results announced below are a selective sampling from the work [**14**] and significantly extend the theory of operators Qu, Pu as previously developed by such authors as De Giorgi [**2**], Moser [**5**], [**6**], Ladyženskaya and Ural'ceva [**3**], [**4**], Serrin [**1**], [**7**], Stampacchia [**8**], [**9**], Aronson [**1**], Trudinger [**10**], [**11**] and others. The present note discusses the operators Qu, although we remark that the theory for Pu is largely derived through extrapolation of the Qu methods. The *local* results for Qu are in general specializations of their extensions to Pu. The proofs of the theorems here, as supplied in [**14**], involve some interesting new test function techniques.

Let us restrict ourselves here to a coefficient structure determined by polynomials in $|u|$ and $|Du|$ with coefficients in L_p spaces. For a more general type of structure see [**13**]. Let Ω be a bounded domain in E^n and $G \subset \Omega \times E^{n+1}$. We define the following structural inequalities, A_1, A_2, B, to be satisfied for all $x, u, p \in G$:

AMS 1970 *subject classifications.* Primary 35B45, 35D10, 35J15, 35J60; Secondary 34J70.

Key words and phrases. Generalized solution, quasilinear differential inequality, elliptic operator, parabolic operator, global estimates, maximum principle, local estimates, weak Harnack inequality, Harnack inequality, Hölder continuity, semicontinuity.