

SOME OPEN QUESTIONS IN THE THEORY OF SINGULARITIES¹

BY OSCAR ZARISKI

ABSTRACT. Three approaches to a theory of equisingularity of complex analytic (or algebraic) hypersurfaces are outlined, based respectively on topology (topological equivalence of embedded varieties), differential geometry (Conditions A and B of Whitney) and algebraic geometry (the author's inductive discriminant criterion). For each of these approaches some unsolved questions and (or) conjectures are formulated, especially in regard to the relationship between these three points of view.

1. If I were giving this address some 7 or 8 years ago, the open question which I would have then given top priority in my talk would have been the problem of reduction of singularities of algebraic varieties of dimension > 3 , over ground fields of characteristic zero, the case of surfaces and of three-dimensional varieties having been settled in some of my earlier work in the late thirties and the early forties, work which also included the solution of the general problem of local uniformization in characteristic zero, in any dimension (see [10], [11], [12] and [13]). For twenty years, after these earlier papers of mine had appeared in print, no further progress was made in the direction of the solution of the problem of reduction of singularities. Personally I felt that I have devoted enough time and effort to that problem, that I needed a change of pace, and have therefore turned to other questions in my field. Some of my mathematical friends believed, no doubt, that during these twenty years I never gave up trying, and it is quite possible that these friends have drawn the—to me flattering—conclusion that since I am not able to prove the general reduction theorem, that theorem must be false. It is even probable that they were greatly tempted to look for and find a counterexample. Fortunately, Hironaka put a stop to this state of affairs by his

¹ Retiring Presidential address delivered before the Seventy-seventh Annual Meeting of the Society in Atlantic City on January 22, 1971; received by the editors February 11, 1971; the author wishes to thank the NSF for support under the Grant No. GP-9667 to Harvard University.

AMS 1970 subject classifications. Primary 14B05, 32C40; Secondary 14H20, 32C45.

Key words and phrases. Singularity, hypersurface, topological equisingularity, differential-geometric equisingularity, algebro-geometric equisingularity, equimultiplicity, blowing-up transformation, equisingularity along a subvariety, Whitney stratification, equisingular projection, equisaturation.