BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 3, May 1971

CLASSIFICATION OF THE SIMPLE SEPARABLE REAL L^*-ALGEBRAS[1]

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Communicated by M. H. Protter, October 20, 1970

A real (complex) L^*-algebra is a Lie algebra L whose underlying vector space is a real (complex) Hilbert space and such that, for each $x \in L$, there exists an $x^* \in L$ satisfying $\langle [x, y], z\rangle = \langle y, [x^*, z]\rangle$ for all y, z in L. J. R. Schue [11], [12] defined and classified the simple separable complex L^*-algebras. V. K. Balachandran [1], [2], [3], [4], [5] gave a more general setting to the techniques used by Schue for not necessarily separable L^*-algebras; he also defined the notions of real form and compact real form.

The main result of this work is the classification of the simple separable real L^*-algebras up to L^*-isomorphisms. The classification was also obtained, independently, by Mr. Pierre de la Harpe.

The following can be shown:

THEOREM 1. *The complexification $\tilde{L}$ of a simple L^*-algebra L is not simple if and only if $L = M^R$, where M is a simple complex L^*-algebra (M^R denotes the real L^*-algebra obtained from M by restriction of scalars).*

Therefore, the classification reduces essentially, aside from simple L^*-algebras having a complex structure which are in a one-to-one correspondence with the simple complex L^*-algebras, to the study of the real forms of all simple complex L^*-algebras.

If L is a real form of a semisimple L^*-algebra $\tilde{L}$, the decomposition $L = K + M$ (Hilbert direct sum), where $K = \{a \in L : a^* = -a\}$ and $M = \{a \in L : a^* = a\}$, defines an involutive L^*-automorphism S of L ($S|K = \mathrm{id}$ and $S|M = -\mathrm{id}$) which can be extended to $\tilde{L}$ by linearity. S is called the involution of $\tilde{L}$ associated to L. Conversely, if S is an involutive L^*-automorphism of $\tilde{L}$, then S leaves the unique compact form U (set of all skew-adjoint elements of $\tilde{L}$) invariant and we have $U = K + iM$, the decomposition of U into eigenspaces of S. The real form $L = K + M$ is said to be associated to S.

There is a one-to-one correspondence between conjugacy classes of

AMS 1970 *subject classifications.* Primary 22E65; Secondary 17B65.

Key words and phrases. L^*-algebras, Cartan subalgebras, Cartan decompositions, Hilbert-Schmidt operators.

[1] These results are contained in the author's doctoral dissertation written under the direction of Professor Ichiro Satake at the University of California, Berkeley.