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C^*-ALGEBRAS GENERATED BY MEASURES[1]

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We announce here some results dealing with nonabelian extensions of the theory of almost periodic functions to the duals of compact groups. For G a locally compact group, let $\hat{G}$ be the dual of G (the set of equivalence classes of continuous, irreducible, unitary representations of G). For $\pi \in \hat{G}$ and $\mu \in M(G)$, the measure algebra of G, let $\pi(\mu)$ be the Fourier-Stieltjes transform of μ at π. Let $\|\mu\|_\infty$ be $\sup\{\|\pi(\mu)\| : \pi \in \hat{G}\}$, and let $\mathfrak{M}(\hat{G})$ be the C^*-completion of $M(G)$ relative to the norm $\|\cdot\|_\infty$. Let $\mathfrak{M}_a(\hat{G})$, $\mathfrak{M}_d(\hat{G})$ be the closures in $\mathfrak{M}(\hat{G})$ of $L^1(G)$ (the space of measures absolutely continuous with respect to left Haar measure), $M_d(G)$ (the space of discrete measures) respectively. The algebra $\mathfrak{M}_d(\hat{G})$ is a nonabelian analogue of the classical algebra of almost periodic functions. A standard reference for C^*-algebras is [1].

We denote the spectrum of $\mathfrak{M}(\hat{G})$ by $\kappa\hat{G}$. In the abelian case this is the closure of the dual group of G in the spectrum of $M(G)$. In general $\hat{G}$ is identified with a dense open subset of $\kappa\hat{G}$ and $\kappa\hat{G}\backslash\hat{G}$ is the annihilator of $\mathfrak{M}_a(\hat{G})$. We investigate the C^*-extension of the canonical projection which maps a measure to its discrete part. This makes possible a proof that $\kappa\hat{G}\backslash\hat{G}$ contains a homeomorphic copy of the reduced dual of G_d, the group G made discrete. We further show that if G is nondiscrete and G_d is amenable then the sup and lim sup norms are identical on $\mathfrak{M}_d(\hat{G})$, and if $\mu \in \mathfrak{M}_d(\hat{G})$ then $\mu \in M_d(\hat{G})$ $(\mu \in M(G))$.

For $S \subset \kappa\hat{G}$ let $\mathfrak{N}(S) = \{\phi \in \mathfrak{M}(\hat{G}) : \pi(\phi) = 0 \text{ for all } \pi \in S\}$. Then $\mathfrak{N}(S)$ is a closed ideal in $\mathfrak{M}(\hat{G})$. Let $\mathfrak{M}(S) = \mathfrak{M}(\hat{G})/\mathfrak{N}(S)$ be the quotient C^*-algebra.

Denote the locally compact group G made discrete by G_d. Then $\hat{G}_d$ is the dual of G_d and is also the spectrum of $\mathfrak{M}(\hat{G}_d) = \mathfrak{M}_a(\hat{G}_d) = \mathfrak{M}_d(\hat{G}_d)$. Each $\pi \in \hat{G}$ gives an irreducible unitary representation of G_d; thus $\hat{G}$ is identified with a subset of $\hat{G}_d$. We denote the closure of $\hat{G}$ in $\hat{G}_d$ by $\hat{G}_{dc}$. Further denote the reduced dual of G_d by $\hat{G}_{dr}$, the set of $\pi \in \hat{G}_d$ which are weakly contained in the left regular representa-

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