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A NOTE ON COBORDISM OF POINCARÉ DUALITY SPACES

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1. **Introduction.** Let Ω_n^{PD} denote the group of cobordism classes of oriented Poincaré duality spaces of dimension n. (See [2] for definitions.) The Pontrjagin-Thom construction yields a natural homomorphism $p:\Omega_n^{\mathrm{PD}} \to \pi_n(MSG)$ where MSG is the Thom spectrum associated to the universal spherical fibration over BSG.

N. Levitt [2] has shown that if $n \not\equiv 3 \pmod 4$, then p is surjective, and if $n \equiv 3 \pmod 4$, then cokernel$(p) \subseteq \boldsymbol{Z}_2$. More precisely, Levitt has shown that, if $n \geqq 3$, there is a subgroup $\bar{\Omega}_n \subseteq \Omega_n^{\mathrm{PD}}$ (it is likely that $\bar{\Omega}_n = \Omega_n^{\mathrm{PD}}$) and an exact sequence

$$(1.0) \qquad \cdots \to P_n \to \bar{\Omega}_n \xrightarrow{p} \pi_n(MSG) \to P_{n-1} \to \cdots$$

where $P_n = \boldsymbol{Z}$, 0, $\boldsymbol{Z}_2$, 0 as $n \equiv 0, 1, 2, 3 \pmod 4$, respectively. Further, image$(P_n) \subset \Omega_n^{\mathrm{PD}}$ is generated by the cobordism class $[K^n]$ where, if $n \equiv 0 \pmod 4$, K^n is the almost parallelizable Milnor manifold of index 8, and, if $n \equiv 2 \pmod 4$, K^n is the almost parallelizable Kervaire manifold constructed by plumbing together the tangent bundles of two $(n/2)$-spheres. (K^4 is not a manifold, but it is a Poincaré duality space.)

Our main results, proved in §2, are the following.

THEOREM 1.1. *The Kervaire manifold, K^{4k+2}, bounds a Poincaré duality space.*

THEOREM 1.2. *The Milnor manifold, K^{4k}, is Poincaré duality cobordant to* $8(\boldsymbol{C}P(2))^k$.

It follows from Theorem 1.1 that the long exact sequence (1.0) contains short exact sequences

$$0 \to \bar{\Omega}_{4k+3} \to \pi_{4k+3}(MSG) \to \boldsymbol{Z}_2 \to 0.$$

Our proof of Theorem 1.1 can be formulated to show that this sequence is actually split exact.

Theorem 1.2 describes the short exact sequences

$$0 \to \boldsymbol{Z} \to \bar{\Omega}_{4k} \to \pi_{4k}(MSG) \to 0$$

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