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EXISTENCE AND UNIQUENESS FOR NONLINEAR NEUTRAL-DIFFERENTIAL EQUATIONS[1]

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ABSTRACT. Fixed point theorems are used to prove existence and uniqueness of the C^1 solution of the initial-value problem for a functional-differential equation of neutral type.

1. **Introduction.** In this paper we consider the initial-value problem (IVP) for the functional-differential equation of neutral type

$$x'(t) = f(t, x(t), x(g(t, x(t))), x'(h(t, x(t)))), \tag{1}$$

with the initial condition

$$x(0) = x_0. \tag{2a}$$

Here $f(t, x, y, z)$, $g(t, x)$ and $h(t, x)$ are continuous functions with $g(0, x_0) = h(0, x_0) = 0$. We assume further that the algebraic equation $z = f(0, x_0, x_0, z)$ has a real root z_0, and we require that

$$x'(0) = z_0. \tag{2b}$$

Existence theorems for IVP's for equation (1) have been proved by R. D. Driver [1] for the case where $h(t, x) < t$, and recently by V. P. Skripnik [2] under the hypotheses that f is sufficiently small, $h(t, x)$ is independent of x, and f is linear in the argument $x'(h(t))$. Our existence theorem requires none of these hypotheses. Under some additional conditions we obtain a local uniqueness theorem, and obtain as a corollary a result on existence of continuous solutions of certain nonlinear functional equations.

2. **Existence.** Let $\alpha > 0$ and let $J = [-\alpha, \alpha]$. We shall make the following assumptions concerning the IVP (1)–(2a)–(2b):

(i) $f(t, x, y, z)$ is continuous in some region in R^4 containing

$$P = \{(t, x, y, z) : |t| \leqq \alpha, |x - x_0| \leqq \beta, |y - x_0| \leqq \beta, |z| \leqq M\}$$

where α, β and $M > |z_0|$ are positive constants. We assume that $\alpha \leqq \beta/M$ and that $\sup_{(t,x,y,z)\in P} |f(t, x, y, z)| \leqq M$.

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