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STRUCTURE OF WITT RINGS, QUOTIENTS OF ABELIAN GROUP RINGS, AND ORDERINGS OF FIELDS

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1. **Introduction.** In 1937 Witt [9] defined a commutative ring $W(F)$ whose elements are equivalence classes of anisotropic quadratic forms over a field F of characteristic not 2. There is also the Witt-Grothendieck ring $WG(F)$ which is generated by equivalence classes of quadratic forms and which maps surjectively onto $W(F)$. These constructions were extended to an arbitrary pro-finite group, $\mathfrak{G}$, in [1] and [6] yielding commutative rings $W(\mathfrak{G})$ and $WG(\mathfrak{G})$. In case $\mathfrak{G}$ is the galois group of a separable algebraic closure of F we have $W(\mathfrak{G})=W(F)$ and $WG(\mathfrak{G})=WG(F)$. All these rings have the form $\boldsymbol{Z}[G]/K$ where G is an abelian group of exponent two and K is an ideal which under any homomorphism of $\boldsymbol{Z}[G]$ to $\boldsymbol{Z}$ is mapped to 0 or $\boldsymbol{Z}2^n$. If C is a connected semilocal commutative ring, the same is true for the Witt ring $W(C)$ and the Witt-Grothendieck ring $WG(C)$ of symmetric bilinear forms over C as defined in [2], and also for the similarly defined rings $W(C, J)$ and $WG(C, J)$ of hermitian forms over C with respect to some involution J.

In [5], Pfister proved certain structure theorems for $W(F)$ using his theory of multiplicative forms. Simpler proofs have been given in [3], [7], [8]. We show that these results depend only on the fact that $W(F)\cong\boldsymbol{Z}[G]/K$, with K as above. Thus we obtain unified proofs for all the Witt and Witt-Grothendieck rings mentioned.

Detailed proofs will appear elsewhere.

2. **Homomorphic images of group rings.** Let G be an abelian torsion group. The characters χ of G correspond bijectively with the homomorphisms ψ_χ of $\boldsymbol{Z}[G]$ into some ring A of algebraic integers generated by roots of unity. (If G has exponent 2, then $A=\boldsymbol{Z}$.) The minimal prime ideals of $\boldsymbol{Z}[G]$ are the kernels of the homomorphisms $\psi_\chi:\boldsymbol{Z}[G]\to A$. The other prime ideals are the inverse images under the ψ_χ of the maximal ideals of A and are maximal.

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