BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

THE ADAMS-NOVIKOV SPECTRAL SEQUENCE FOR THE SPHERES

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Communicated by P. E. Thomas, June 17, 1970

The Adams spectral sequence has been an important tool in research on the stable homotopy of the spheres. In this note we outline new information about a variant of the Adams sequence which was introduced by Novikov [7]. We develop simplified techniques of computation which allow us to discover vanishing lines and periodicity near the edge of the E_2-term, interesting elements in $E_2^{2,*}$, and a counterexample to one of Novikov's conjectures. In this way we obtain independently the values of many low-dimensional stems up to group extension. The new methods stem from a deeper understanding of the Brown-Peterson cohomology theory, due largely to Quillen [8]; see also [4]. Details will appear elsewhere; or see [11].

When p is odd, the p-primary part of the Novikov sequence behaves nicely in comparison with the ordinary Adams sequence. Computing the E_2-term seems to be as easy, and the Novikov sequence has many fewer nonzero differentials (in stems $\leqq 45$, at least, if $p=3$), and periodicity near the edge. The case $p=2$ is sharply different. Computing E_2 is more difficult. There are also hordes of nonzero differentials d_3, but they form a regular pattern, and no nonzero differentials outside the pattern have been found. Thus the diagram of E_4 ($=E_\infty$ in dimensions $\leqq 17$) suggests a vanishing line for E_∞ much lower than that of E_2 of the classical Adams spectral sequence [3].

It is a pleasure to thank Arunas Liulevicius, my thesis adviser, for his help. In particular, parts of the proofs of Proposition 1 and Theorem 7 are due to him. I am also grateful to many others for their suggestions, and especially to Frank Adams.

1. The spectral sequence. The construction of the classical Adams spectral sequence for the spheres [1] works equally well if the spec-

AMS 1970 *subject classifications.* Primary 55H15, 55G25, 55E45; Secondary 55B20.

Key words and phrases. Stable homotopy of spheres, Adams spectral sequence, unitary cobordism, Brown-Peterson spectrum, projective resolution, Arf invariant, vanishing line, edge periodicity.

[1] Most of the results announced in this paper were in the author's doctoral dissertation, written under an NSF graduate fellowship at the University of Chicago.