BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

ISOMORPHISM THEORY OF CONGRUENCE GROUPS

BY ROBERT E. SOLAZZI

Communicated by I. N. Herstein, May 18, 1970

This note is an announcement of four theorems I proved in [5]–[9] on the isomorphisms of the linear, symplectic, and unitary congruence groups. A sketch of the proofs is given.

NOTATION. Let V be an n-dimensional vector space over the field F, $n \geqq 2$. For $\sigma \in GL_n(V)$, let $\check{\sigma}$ denote the contragredient of σ (inverse of the transpose). Let $^-$ denote the natural map of $GL_n(V)$ onto $PGL_n(V)$; for any subset S of $GL_n(V)$, $\bar{S}$ is the image of S in $PGL_n(V)$ under the $^-$ map.

A transvection τ is a linear transformation of determinant one which fixes all vectors of some hyperplane, called the proper hyperplane of τ. If $\tau \neq 1$ then $(\tau - 1)V$ is a line called the proper line of τ. An element $\bar{\tau}$ of $PGL_n(V)$ is called a (projective) transvection if τ is a transvection. The proper line and proper hyperplane of $\bar{\tau}$ are defined as the proper line and hyperplane of τ.

Congruence groups. Again let V be an n-dimensional vector space over F, $n \geqq 2$; let $\mathfrak{o}$ be any integral domain with quotient field F. An $\mathfrak{o}$-module $M \subset V$ is *bounded* if there is an $\mathfrak{o}$-linear isomorphism of M into some free $\mathfrak{o}$-module of finite dimension. Assume M is a bounded $\mathfrak{o}$-module with $FM = V$. We define the integral linear group $GL_n(M)$ as $\{\sigma \in GL_n(V) \mid \sigma M = M\}$. The integral symplectic group $Sp_n(M)$ is $\{\sigma \in Sp_n(V) \mid \sigma M = M\}$ and is only defined for even n.

Let $f(x, y)$ be a nondegenerate hermitian form on V whose involution maps the domain onto itself; the integral unitary group $U_n(M, f)$ is defined as $U_n(M, f) = \{\sigma \in U_n(V, f) \mid \sigma M = M\}$.

Now let $\mathfrak{a}$ be a nonzero ideal in $\mathfrak{o}$. Consider the groups

$$GL_n(M; \mathfrak{a}) = \{\sigma \in GL_n(M) \mid (\sigma - 1)M \subset \mathfrak{a}M\},$$
$$SL_n(M; \mathfrak{a}) = GL_n(M; \mathfrak{a}) \cap SL_n(V)$$

and let $TL_n(M; \mathfrak{a})$ be the group generated by all transvections in $GL_n(M; \mathfrak{a})$.

Now put $Sp_n(M; \mathfrak{a}) = \{\sigma \in Sp_n(M) \mid (\sigma - 1)M \subset \mathfrak{a}M\}$ and let

---

AMS 1970 *subject classifications*. Primary 20H05, 15A63; Secondary 15A57.

Key words and phrases. Linear group, symplectic, unitary group, congruence group, transvection, semisimilitute, isotropic line, collineation.