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DIFFEOMORPHISMS OF MANIFOLDS AND SEMIFREE ACTIONS ON HOMOTOPY SPHERES

BY WILLIAM BROWDER AND TED PETRIE

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The paper of which this is an announcement is naturally divided into two parts. One is the study of the group of pseudoisotopy classes of diffeomorphisms of a particular class of manifolds. The second is the use of the results of this study to construct and classify the "group" of semifree H actions on homotopy $2k+q+1$ spheres with a homotopy q sphere as fixed point set. Here H is a compact Lie Group and a semifree H action on a manifold M is a smooth action on M such that the action is free off the fixed point set of H. The class of groups which can act semifreely on any manifold is restrictive but rich and this class has been determined. It coincides with the set of groups which have a real representation such that no element of the group other than the identity has one as an eigenvalue. Equivalently, it is the set of groups which can act freely and linearly on a standard k-sphere S^k.

Particularly interesting consequences of our study are these: If H acts freely on S^k, let $P^k=S^k/H$ be the orbit space and $D(P^{k,q}, \partial)$ be the group of pseudoisotopy classes rel boundary of diffeomorphisms of $P^k\times D^q$ which are the identity on $P^k\times S^{q-1}$. If H is Z_p, p odd, S^0, S^1, S^3 or N the normalizer of S^1 in S^3, we can give a complete description of the torsion free part of $D(P^{k,q}, \partial)$. For other groups H, if k is sufficiently large compared with q, we can determine this group modulo knowledge of the Wall surgery obstruction group

$$L_n^s(\pi_0(H), \omega(H)), \qquad n = k + q - \dim H + 1.$$

Let $\mathcal{A}^{k,q}=$ the set of isomorphism classes of semifree H actions on homotopy $(k+q+1)$-spheres with a homotopy q-sphere as fixed point set. We show that a certain subset $\mathcal{A}_0^{k,q}\subset\mathcal{A}^{k,q}$ has the natural structure of an abelian group. We exhibit an epimorphism of groups $D(P^{k,q}, \partial)\to\mathcal{A}_0^{k,q}$ and determine the torsion free part of the group $\mathcal{A}_0^{k,q}$ in certain cases. In particular, we settle a question of [1] showing that there are only a finite number of inequivalent semifree S^1

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