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ON THE DEMIREGULARITY OF WEAK SOLUTIONS OF NONLINEAR ELLIPTIC EQUATIONS

BY JINDŘICH NEČAS

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1. **Introduction.** Let Ω be a bounded domain with infinitely differentiable boundary $\partial\Omega$ in n-dimensional real space R_n. Let k be a positive integer, and let us define the functions $a_i(x, \xi)$ for multi-indices $|i| = i_1 + i_2 + \cdots + i_n \leqq k$, continuous in $\bar{\Omega} \times R_\kappa$, where κ is the number of indices of length $\leqq k$. By $W_p^{(k)}(\Omega)$, we denote the Sobolev space of L_p-functions whose derivatives up to the order k are also L_p-functions, with the norm

$$\|u\|_{k,p} = \left(\int_\Omega \sum_{|i| \leq k} | D^i u |^p dx \right)^{1/p},$$

where the usual notation

$$D^i = \frac{\partial^{|i|}}{\partial x_1^{i_1} \cdots \partial x_n^{i_n}}$$

is introduced. The functions $a_i(x, \xi)$ are supposed to satisfy the growth-conditions:

(1.1) $|a_i(x, \xi)| \leqq c(1 + |\xi|)$.

Let functions $u_0 \in W_2^{(k)}(\Omega)$ and $f_i \in L_2(\Omega)$, $|i| \leqq k$, be given. Let $\mathring{W}_p^{(k)}(\Omega)$ be the closure of $D(\Omega)$, the space of infinitely differentiable functions with compact support, in the space $W_p^{(k)}(\Omega)$.

A function u from $W_2^{(k)}(\Omega)$ is called a weak solution of the Dirichlet problem: $\partial^l u/\partial n^l = \partial^l u_0/\partial n^l$ on $\partial\Omega$, $l = 0, 1, \cdots, k-1$, (where $\partial/\partial n$ is the derivative with respect to the outer normal),

$$\sum_{|i| \leq k} (-1)^{|i|} D^i(a_i(x, \xi(u))) = \sum_{|i| \leq k} (-1)^{|i|} D^i f_i \qquad \text{in } \Omega$$

(where the components of $\xi(u)$ are $D^i u$) if

(1.2) $u - u_0 \in \mathring{W}_2^{(k)}(\Omega)$,

(1.3) for every v in $\mathring{W}_2^{(k)}(\Omega)$:

$$\int_\Omega \sum_{|i| \leq k} D^i v a_i(x, \xi(u)) dx = \int_\Omega \sum_{|i| \leq k} D^i v f_i dx.$$