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DISCOUNTED AND POSITIVE STOCHASTIC GAMES

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1. **Introduction.** The main purpose of this note is to announce a few results on stochastic games. A stochastic game is determined by five objects: S, A, B, q and r. S, A and B are nonempty Borel Subsets of Polish spaces and r is a bounded measurable function on $S \times A \times B$. We interpret S as the state space of some system and A, B as the set of actions available to players I and II respectively at each state. When the system is in state s and players I and II choose action a and b respectively, the system moves to a new state according to the distribution $q(\cdot \mid s, a, b)$ and I receives from II, $r(s, a, b)$ units of money. Then the whole process is repeated from the new state s'. The problem, then, is to maximize player I's expected income as the game proceeds over the infinite future and to minimize player II's expected loss.

A strategy π for player I is a sequence $\pi_1, \pi_2, \cdots$, where π_n specifies the action to be chosen by player I on the nth day by associating (Borel measurably) with each history

$$h = (s_1, a_1, b_1, \cdots, s_{n-1}, a_{n-1}, b_{n-1}, s_n)$$

of the system a probability distribution $\pi_n(\cdot \mid h)$ on the Borel sets of A. Call π a *stationary strategy* if there is a Borel map f from S to P_A, where P_A is the set of all probability measures on the Borel sets of A, such that $\pi_n = f$ for each $n \geqq 1$ and in this case, π is denoted by $f^{(\infty)}$. Strategies and stationary strategies are defined similarly for II.

Let β be any fixed nonnegative number satisfying $0 \leqq \beta < 1$. A pair (π, Γ) of strategies for I and II associates with each initial state s, a nth day expected income $r_n(\pi, \Gamma)(s)$ for I and a total expected discounted income

$$I_\beta(\pi, \Gamma)(s) = \sum_{n=1}^{\infty} \beta^{n-1} r_n(\pi, \Gamma)(s).$$

Such stochastic games are called discounted stochastic games. Positive stochastic games are those where $r(s, a, b) \geqq 0$ $\forall s, a, b$ and $\beta = 1$.

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