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KOEBE SETS FOR UNIVALENT FUNCTIONS WITH TWO PREASSIGNED VALUES

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1. **Introduction.** Let $\mathfrak{M}_M$ denote the set of all functions $f(z)$ that are analytic and univalent in the unit disc Δ and satisfy the conditions $f(0)=0$, $f(z_0)=z_0$, and $|f(z)|\leqq M$, where z_0 is a fixed point of Δ, $z_0\neq 0$, and where M is fixed, $1<M\leqq\infty$.

Although the class $\mathfrak{M}_\infty$ has been a popular one to study, very little seems to have been done with $\mathfrak{M}_M$. We aim to correct this oversight by beginning a study of $\mathfrak{M}_M$. In this paper we obtain the exact value of the "Koebe constant" for $\mathfrak{M}_M$ and we determine the Koebe sets for

(i) the set $\mathfrak{M}_M^*$ consisting of those elements $f(z)$ of $\mathfrak{M}_M$ for which $f(\Delta)$ is starlike with respect to the origin, and

(ii) the set $\mathfrak{M}_\infty^\alpha$ consisting of those members $f(z)$ of $\mathfrak{M}_\infty$ for which $f(\Delta)$ is convex in the direction $e^{i\alpha}$.

2. **Main results.** By the *Koebe constant* for $\mathfrak{M}_M$ we mean the radius of the largest disc, center at the origin, that lies in the set $\bigcap[f(\Delta)\,|\,f\in\mathfrak{M}_M]$, the Koebe set for $\mathfrak{M}_M$.

THEOREM 1. *The Koebe constant for $\mathfrak{M}_M$ is given by*

$$r(\mathfrak{M}_M) = 2\delta^2 - M - 2\delta(\delta^2 - M)^{1/2}, \tag{1}$$
$$\delta = \frac{M-|z_0|}{1-|z_0|}.$$

This result is sharp.

PROOF. First, there is no loss of generality here if z_0 is taken to be real and positive. Hence we set $z_0=r_0>0$. Now we obtain the domain Ω^* from the domain $\Omega\equiv f(\Delta)$ by a circular symmetrization with respect to the half-line $[0, r_0, \infty)$. The domain Ω^* contains the origin

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