BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 1, January 1971

A SHORT PROOF OF A THEOREM OF PLANS ON THE HOMOLOGY OF THE BRANCHED CYCLIC COVERINGS OF A KNOT

BY C. McA. GORDON

Communicated by N. E. Steenrod, July 6, 1970

Let $K \subset S^3$ be a (tame) knot, with complement $C = S^3 - K$, and let $\tilde{C}$ be the infinite cyclic covering of K, i.e. the covering of C corresponding to the commutator subgroup of $\pi_1(C)$. The group of covering translations of $\tilde{C}$ is $H_1(C)$, which is infinite cyclic by Alexander duality; this gives an action of $\boldsymbol{Z}$ on $H_1(\tilde{C})$, and so $H_1(\tilde{C})$ becomes a Λ-module, where Λ is the integral group ring of $\boldsymbol{Z}$. We identify Λ with the ring of polynomials in a single variable t, (positive and negative powers of t being allowed), with integral coefficients. (See [4].)

The k-fold branched cyclic covering of K, M_k $(k \geqq 1)$ is defined by taking the covering of C corresponding to the kernel of the composition:

$$\pi_1(C) \to H_1(C) \cong \boldsymbol{Z} \to \boldsymbol{Z}_k,$$

and branching about K. (For more details, see [1], [4].) M_k is a closed, orientable 3-manifold: for example, M_1 is just S^3.

If $M(t) = (m_{ij}(t))$, $m_{ij}(t) \in \Lambda$, is a presentation matrix for $H_1(\tilde{C})$ as a Λ-module, then it can be shown that a presentation matrix for $H_1(M_k)$ $(k > 1)$ as an abelian group is obtained by substituting for each entry $m_{ij}(t)$, which is some finite formal sum, $\sum_\nu \alpha_\nu t^\nu$, say, the $k \times k$ block $\sum_\nu \alpha_\nu T_k^\nu$, where the summation indicates ordinary matrix addition, and T_k is the $k \times k$ matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 \\ 0 & 0 & 0 & 1 \cdots 0 \\ \vdots & & & \ddots \;\; \vdots \\ 0 & 0 & 0 & 0 \cdots 1 \\ 1 & 0 & 0 & 0 \cdots 0 \end{pmatrix},$$

AMS 1970 *subject classifications*. Primary 55A10, 55A25.

Key words and phrases. Classical knot theory, branched cyclic covering, Seifert matrix, Plans' Theorem, direct double.