

BOUNDED CONVERGENCE OF ANALYTIC FUNCTIONS

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ABSTRACT. This is a survey of one part of the area of interaction between complex analysis and functional analysis. The space of bounded analytic functions on an open set is considered under numerous topologies, unified by the theme of bounded pointwise convergence. Related problems in approximation theory, the structure of ideals of bounded analytic functions, and inner and outer (also interior and exterior) functions are discussed.

I will give a prejudiced survey of some recent developments in the field of interaction between complex analysis and functional analysis. I will also pose a number of unsolved problems. The real measure of the mathematics I will discuss, in my opinion, is the problems it raises and the problems it solves in the hard analysis of bounded analytic functions. The basis of the work at hand is found in my 1966 paper with Shields [23] and in my paper with Ryff [21] that has appeared since this lecture was given. The central theme is the bounded convergence of a sequence of bounded analytic functions.

Let G be an open set in the complex plane, and let $B_H(G)$ be the space of all bounded analytic functions on G . If $\{f_n\}$ is a sequence of functions in $B_H(G)$, we say that f_n converges boundedly to f , and write $f_n \xrightarrow{B} f$, to mean that

- (i) $\{f_n(z)\}$ is uniformly bounded in G , and

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