GIRTHS AND FLAT BANACH SPACES

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J. J. Schäffer [3] introduced an interesting parameter for normed linear spaces. It is termed *girth* and is the infimum of the lengths of all centrally symmetric simple closed rectifiable curves which lie in the boundary of the unit ball. More precisely, let X be a Banach space with norm denoted by $\|\cdot\|$ and with dim $X \geqq 2$. A *curve* in X will be a *rectifiable geometric curve* defined by Busemann [1] as the equivalence class of curves (i.e., continuous functions from a compact interval of real numbers into the space X with the metric given by $\|\cdot\|$) which have the same standard representation in terms of arc-length. Given a curve c we denote its *length* by $\lambda(c)$; and we denote by $\gamma_c(s)$, $0 \leqq s \leqq \lambda(c)$, its *standard representation in terms of arc-length.* We say that $\gamma_c(0)$ and $\gamma_c(\lambda(c))$ are the *initial* and *final points* of c. A curve c is *simple* if γ_c is injective. Following common usage, a curve c often stands for the common range of its parametrizations, which is a compact subset of X. Thus we say, for example, "$x \in c$," or "the linear hull of c," or "c lies in a subset A of X," etc.

Let B denote the unit ball of X and S the unit sphere. As usual the *inner metric* δ *of* S is defined for all $x, y \in S$ by

$$\delta(x, y) = \inf\{\lambda(c) : c \text{ a curve lying in } S \text{ having } x \text{ and } y \text{ for its initial and final points}\}.$$

The *girth of* B, denoted by girth(B), is defined by

$$\operatorname{girth}(B) = 2 \inf\{\delta(x, -x) : x \in S\};$$

equivalently, (cf. [3])

$$\operatorname{girth}(B) = \inf\{\lambda(c) : c \text{ a simple closed curve lying in } S,\ c \text{ centrally symmetric}\},$$

where c is said to be *centrally symmetric* if $\gamma_c(s) = \gamma_c(s + \lambda(c)/2)$, for

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