RESTRICTION TO A PARABOLIC SUBGROUP AND IRREDUCIBILITY OF DEGENERATE PRINCIPAL SERIES OF Sp(2, C)

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A series of unitary representations of a semisimple Lie group is said to be a *principal series* if the representations are induced from characters of a so-called parabolic subgroup. In the event that the parabolic subgroup is minimal (i.e., a Borel subgroup) one says that the series is *nondegenerate.* It has recently been shown through the efforts of Kostant and Wallach, among others, that for the complex semisimple groups all of the representations in the nondegenerate principal series are irreducible. However, at present no such general result is known for the remaining series; i.e., the *degenerate principal series.*

This note considers the degenerate principal series of the rank two complex symplectic group $G=\mathrm{Sp}(2, \boldsymbol{C})$. G has exactly two such series, each associated with a maximal parabolic subgroup which turns out to be a semidirect product of a normal nilpotent group with a subgroup isomorphic to the 2×2 complex general linear group $\mathrm{GL}(2, \boldsymbol{C})$. In one case the nilpotent group is abelian, and quite elementary classical Fourier analysis leads to a proof that the representations in the corresponding degenerate series are already irreducible when restricted to the parabolic subgroup. For the other case the analysis is much different, since the nilpotent group is nonabelian and the restriction is not irreducible.

In what follows we investigate this latter situation. In particular, we develop the required nonabelian harmonic analysis, use it to decompose the restrictions of the representations, and show that the representations of G in this degenerate principal series are indeed *not all* irreducible. In this context one also obtains a natural variant of the Shale-Weil representation for the complex rank two special linear group $\mathrm{SL}(2, \boldsymbol{C})$.

In a paper in preparation we extend these results to the rank n real and complex case and develop for these degenerate principal

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