ONE-PARAMETER SEMIGROUPS OF ISOMETRIES

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Let $t \to V_t$ for $t \geqq 0$ be a strongly continuous one-parameter semigroup of isometries on a Hilbert space H. The easiest example of such a semigroup which is not unitary is given by considering the Hilbert space $\tilde{H} = L^2(0, \infty)$ consisting of those Lebesgue square-integrable functions on $(-\infty, \infty)$ which are supported on $(0, \infty)$. On $\tilde{H}$, we consider the (nonunitary) isometries

$$(T_t f)(x) = f(x - t).$$

Recently, the C^*-algebra $\mathfrak{A}(T_t : t \geqq 0)$ generated by the semigroup $t \to T_t$ has been studied in detail [2], [3], [4].

In this note, we show that for any strongly continuous one-parameter semigroup of isometries $t \to V_t$ with V_{t_0} not unitary for some t_0, $\mathfrak{A}(V_t : t \geqq 0)$ is *-isomorphic with $\mathfrak{A}(T_t : t \geqq 0)$. The proof is modelled after the corresponding result for C^*-algebras generated by a single isometry [1].

The main fact that we use is a generalization due to Cooper [6, p. 142] of the Wold decomposition of a single isometry [5, p. 109]. This generalization states that for $t \to V_t$, $t \geqq 0$, a strongly continuous one-parameter semigroup of isometries on H, there is a Hilbert space K with a strongly continuous one-parameter unitary semigroup $t \to U_t$ on K, there is a cardinal α, and there is an isometry U from H onto $K \oplus \tilde{H} \oplus \cdots \oplus \tilde{H} \oplus \cdots$ where $\tilde{H}$ occurs with multiplicity α, such that

$$UV_tU^* = U_t \oplus T_t \oplus \cdots \oplus T_t \oplus \cdots .$$

The multiplicity α is a unitary invariant which can be read off from the infinitesimal generator of $t \to V_t$ [6, p. 142].

In case $K = \{0\}$, we say that $t \to V_t$ is *purely nonunitary* [6, p. 136]. For such semigroups, the multiplicity α is the *only* unitary invariant. A very general way of generating such semigroups is to consider for any measure $d\mu$ which is positive, of bounded variation, and singular with respect to Lebesgue measure on the unit circle T, the singular inner functions [5, p. 66] $\phi_t^\mu(e^{i\theta})$ which are the boundary values of

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