A MEASURABLE MAP WITH ANALYTIC DOMAIN AND METRIZABLE RANGE IS QUOTIENT[1]

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The aim of this note is to prove the statement in the title which is the natural generalization of the classical theorem of N. Lusin for separable metrizable spaces; for historical remarks and classical proof see K. Kuratowski [9, §28].

If P is a topological space we let Baire (P) denote the set P endowed with the σ-algebra of all Baire sets in P. Recall that the collection of Baire sets in P is the smallest σ-algebra of sets such that each real valued continuous function is measurable. A mapping $f:P\to Q$ of topological spaces is called Baire measurable or simply measurable, if $f:\text{Baire}(P)\to\text{Baire}(Q)$ is measurable. A mapping $f:P\to Q$ of measurable spaces is called quotient if f is surjective measurable mapping such that $X\subset Q$ is measurable if $f^{-1}[X]$ is measurable. Now we are prepared to state our main result; the reader may also read an interesting corollary in Theorem 9 below.

THEOREM 1. *Let f be a Baire measurable mapping of an analytic topological space A into a metrizable space M. Then the graph ρ of f, and $Q=f[P]$, are analytic, and the mapping $f:A\to Q$ is a measurable quotient mapping.*

It should be remarked that Theorem 1 is highly nontrivial, and that we need the whole machinary of analytic spaces theory for the proof. Recall that a separated space A is called analytic if there exists an upper semicontinuous compact valued (abbreviated to usco-compact) correspondence of the space Σ of irrational numbers onto A. Thus for completely regular spaces the analytic spaces are just the K-analytic spaces introduced by G. Choquet [2], [3]. In this note we will work in the class of all *completely regular spaces*, and the reader familiar with [6] will observe immediately that Theorem 1 holds for analytic spaces as defined in [6] for general topological spaces. For the convenience of the reader we summarize all requisite facts about analytic spaces.

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