A CHARACTERIZATION OF CONWAY'S GROUP .3[1]

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Statement of result. Let G_0 be the group .3 discovered by J. H. Conway [1], and let s_0 be an involution in the center of a Sylow 2-subgroup of G_0. A direct examination of G_0 shows $(C_{G_0}(s_0)/\langle s_0\rangle \simeq \mathrm{Sp}(6, 2)$, a nonsplit extension.

THEOREM. *Let G be a finite group and s an involution in G, such that $C_G(s)\simeq C_{G_0}(s_0)$. Assume $G\neq C_G(s)O(G)$ ($O(G)$ is the maximal normal odd order subgroup of G). Then $G\simeq G_0$. In particular, G has the following properties*:

(i) *G has order $2^{10}\cdot 3^7\cdot 5^3\cdot 7\cdot 11\cdot 23$, and is simple.*

(ii) *G has two conjugacy classes of involutions. One class is represented by the involution s. A representative t of the second class has centralizer $C_G(t)\simeq\langle t\rangle\times M_{12}$ (M_{12} is the Mathieu group).*

(iii) *The normalizer of a Sylow 23-subgroup is a Frobenius group of order $11\cdot 23$.*

(iv) *The normalizer of a Sylow 11-subgroup is a direct product of Z_2 (the group of order 2) and a Frobenius group of order $5\cdot 11$.*

(v) *The normalizer of a Sylow 7-subgroup is a direct product of Sym_3 (the symmetric group) and a Frobenius group of order $6\cdot 7$ with kernel of order 7.*

(vi) *A Sylow 5-subgroup is nonabelian of exponent 5. There are two classes of elements of order 5, with centralizers of orders $2^2\cdot 3\cdot 5^3$ and $2^2\cdot 3\cdot 5^2$. The normalizer of a Sylow 5-subgroup has order $2^4\cdot 3\cdot 5^3$.*

(vii) *G has no outer automorphisms.*

The character table of G is obtained in the course of the proof, and will appear elsewhere with details of the proof.

Outline of proof. Property (ii) is proved using group-theoretic methods and Wong's characterization of M_{12} [2]. The order follows using a formula of Thompson's requiring only knowledge of the centralizers of involutions and conjugacy of involutions. Properties (iii)–(vi) are then straightforward. This provides sufficient informa-

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