FOLIATIONS AND NONCOMPACT TRANSFORMATION GROUPS

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Introduction. Let G be a Lie group and M a compact C^∞ manifold. In [2] Anosov actions of G on M are defined and proved to be structurally stable.

In this announcement we are concerned with the foliation $\mathfrak{F}$ of M defined by the orbits of G. Under the assumption that G is connected, $\mathfrak{F}$ is C^1 stable (3). If G is connected and nilpotent, G has a compact orbit (4). If G is merely solvable, however, there may be no compact orbit. In fact it can happen that no foliation C^0 close to $\mathfrak{F}$ has a compact leaf (8). Upper bounds for the number of compact orbits of given type are found (9). In (7) we discuss the intersection of certain nilpotent subgroups of a Lie group S with conjugates of a uniform discrete subgroup of S.

Hyperbolic automorphisms of foliations. A *k-foliation* $\mathfrak{F}$ of M is a function assigning to each $x \in M$ the image $\mathfrak{F}_x$ of a C^2 injective immersion $V_x \to M$ of a connected k-dimensional manifold V_x. We require that the *leaf* $\mathfrak{F}_x$ contain x, and that the function $T\mathfrak{F}: M \to G_k(M)$ assigning to $x \in M$ the tangent plane to V_x at x be C^1; here $G_k(M)$ is the manifold of k-planes tangent to M. Equivalently, $T\mathfrak{F}$ is a completely integrable C^1 field of k-planes, and $\mathfrak{F}_x$ is the maximal integral submanifold through x. Thus $\{\mathfrak{F}_x\}_{x \in M}$ is a partition of M. The set $\boldsymbol{F}_k(M)$ of all k-foliations of M inherits the C^0 and C^1 topologies from the set of C^1 maps $M \to G_k(M)$. We also use $T\mathfrak{F}$ to denote the bundle of k-planes tangent to the leaves.

If $\mathfrak{F}$, $\mathcal{G} \in \boldsymbol{F}_k(M)$, a *homeomorphism* $h: \mathfrak{F} \to \mathcal{G}$ is a homeomorphism of M taking each leaf of $\mathfrak{F}$ onto a leaf of $\mathcal{G}$. We call $\mathfrak{F}$ C^1 *stable* if it has a C^1 neighborhood $N \subset \boldsymbol{F}_k(M)$ of foliations homeomorphic to $\mathfrak{F}$.

An *automorphism* g of $\mathfrak{F}$ is a C^1 diffeomorphism of M which is a homeomorphism $\mathfrak{F} \to \mathfrak{F}$. We call g *hyperbolic* if there exists a splitting $TM = E_+ \oplus E_- \oplus T\mathfrak{F}$ invariant under Tg, and such that the following condition holds. For some (and hence any) Riemannian metric on M there exist constants $0 < \lambda < 1 < \mu$ and $n \in \boldsymbol{Z}_+$ such that if $X \in TM$ and $X \neq 0$, then

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