INVARIANT MANIFOLDS

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0. **Introduction.** Let M be a finite dimensional Riemann manifold without boundary. Kupka [5], Sacker [9], and others have studied perturbations of a flow or diffeomorphism of M leaving invariant a compact submanifold. Anosov [2] considers perturbations of a nonsingular flow, which of course leaves invariant each leaf of the foliation by trajectories. In both problems there is an assumption of hyperbolicity in planes normal to the submanifold, or trajectories.

We present a more general theory of diffeomorphisms hyperbolic to a compact laminated subset $\Lambda \subset M$. (This includes flows, by considering the time one map.) We suppose Λ is the disjoint union of injectively immersed submanifolds, called leaves, whose tangent planes vary continuously on Λ. The diffeomorphism is assumed to permute the leaves, and its differential is more hyperbolic normal to the leaves than tangent to them. The main theorems assert that this situation persists under small perturbations of the diffeomorphisms. By means of the technical device of unwinding the leaves of Λ, the proofs are reduced to the case of a single invariant, closed submanifold.

Applications are made to stability of group actions and Ω-stability. See also references [4] through [8].

1. **Definition of r-hyperbolicity.** Let $E \to B$ be a vector bundle with a Finsler structure, and $T: E \to E$ a linear bundle map covering $\phi: B \to B$. Define

$$\rho(T) = \limsup_{n \to \infty} \sup_{x \in B} \|(T^n)_x\|^{1/n}.$$

Let $L \subset E$ be a vector subbundle invariant under T. Call T *r-normally hyperbolic* (or simply r-hyperbolic) *to* L, $r \in \boldsymbol{Z}_+$, if T is a homeomorphism, and there is a splitting $E = N_+ \oplus L \oplus N_-$ invariant under T, such that

$$\rho(T \mid N_-) < \min\{1, \rho(T \mid L)^r\}$$

and

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