NONLINEAR ELLIPTIC BOUNDARY VALUE PROBLEMS 
AND THE GENERALIZED TOPOLOGICAL DEGREE 
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Introduction. It is our purpose in the present note to present a general existence theorem for noncoercive elliptic boundary value problems for operators of the form:

\[ A(u) = \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha A_\alpha(x, u, \ldots, D^m u), \]

on closed subspaces \( V \) of the Sobolev space \( W^{m,p}(G) \), \( G \) an open subset of \( \mathbb{R}^n \), \( n \geq 1 \). This existence theorem is based upon an extension of the theory of the generalized topological degree for \( A \)-proper mappings of Banach spaces introduced in Browder-Petryshyn [8], [9], and, in particular, on an extension of the Borsuk-Ulam theorem to pseudomonotone mappings \( T \) from a reflexive separable Banach space \( V \) to its conjugate space \( V^* \).

To make a precise statement of our general existence theorem possible, we introduce the following notation: For a given \( m \geq 1 \), we let \( \xi \) be the \( m \)-jet of a function \( u \) from \( \mathbb{R}^n \) to \( \mathbb{R}^s \) for some given \( s \geq 1 \), i.e. \( \xi = \{ \xi_\alpha : |\alpha| \leq m \} \), and set

\[ \xi = \{ \xi_\alpha : |\alpha| = m \}, \quad \eta = \{ \eta_\beta : |\beta| \leq m - 1 \}, \]

where each \( \xi_\alpha, \xi_\alpha, \) and \( \eta_\beta \) is an element of \( \mathbb{R}^s \). The set of all \( \xi \) of the above form is an Euclidean space \( \mathbb{R}^{m^s} \), and correspondingly, \( \xi \in \mathbb{R}^{m^s} \), \( \eta \in \mathbb{R}^{m^{s-1}} \).

For each \( \alpha \), \( A_\alpha \) is assumed to be a function from \( G \times \mathbb{R}^m \) to \( \mathbb{R}^s \) satisfying the following conditions:

Assumptions on \( A(u) : (1) A_\alpha(x, \xi) \) is measurable in \( x \) for fixed \( \xi \) and continuous in \( \xi \) for fixed \( x \). For a given \( p \) with \( 1 < p < \infty \), there exists a constant \( c \) such that

\[ |A_\alpha(x, \xi)| \leq c \left( 1 + \sum_{|\beta| \leq m} |\xi_\beta|^{p_\alpha} \right) \]

with \( p_\alpha \leq (p - 1) \) for \( |\alpha| = |\beta| = m \), and

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