## RESEARCH PROBLEMS

The Research Problems department of the *Bulletin* has been discontinued. This final offering consists of recently rediscovered problems and solutions submitted before the closing of the department.

### PROBLEMS.

1. Richard Bellman. *Orthogonal series*

Let $\{u_n(x)\}$ be an orthonormal sequence over the interval $[a, b]$ with the continuous, positive weight function $p(x)$. Let $f(x)$ be a continuous positive function in $[a, b]$ and let $f(x) \sim \sum_{n=1}^{N} a_n u_n(x)$. Does there exist a summability matrix $(s_{nm})$ such that $\sigma_N = \sum_{n=1}^{N} s_{nN} a_n u_n(x)$ converges to $f(x)$ as $N \to \infty$ and $\sigma_N(x) \geqq 0$ for $N \geqq 1$, $a \leqq x \leqq b$?

2. Richard Bellman. *Differential equations*

Let $m_i$, $i=1, 2, \cdots, N$ be moments of a distribution, i.e., $m_i = \int_a^b x^i dG(x)$, $dG \geqq 0$. Consider the linear system $dx_i/dt = \sum_{j=1}^{N} a_{ij} x_j$, $x_i(0) = m_i$. What are necessary and sufficient conditions on the matrix $A = (a_{ij})$ so that the $x_i(t)$, $i = 1, 2, \cdots, N$, are moments for $t \geqq 0$?

3. Richard Bellman. *Approximation of functions*

Let $k(x, y)$ be a continuous function of $x$ and $y$ in the square $0 \leqq x, y \leqq 1$. Determine the minimum of $J(f, g) = \int_0^1 f(x)dx + \int_0^1 g(y)dy$, $k(x, y) \leqq f(x) + g(y)$. Consider the case where $k(x, y)$ is symmetric in $x$ and $y$, and we ask that $f = g$.

Generalize both to the multidimensional case where

$$k(x_1, x_2, \cdots, x_N) \leqq f(x_1, x_2, \cdots, x_k) + g(x_{k+1}, x_{k+2}, \cdots, x_N),$$

and to the case where $J(f, g)$ is a more general functional.

Consider the case where $k(x_1, x_2, x_3)$ is symmetric in $x_1, x_2, x_3$, and we ask for the minimum of $\int_0^1 \int f(x_1, x_2) dx_1 dx_2$ subject to the condition that $k(x_1, x_2, x_3) \leqq f(x_1, x_2) + f(x_1, x_3) + f(x_3, x_1)$ and $f(x_1, x_2)$ is symmetric in $x_1$ and $x_2$.

Is there a systematic procedure for solving problems of this type involving the minimization of $J(g)$ where $f(p) \leqq g(p)$ and $g$ is invariant under a group of operations?

4. George Brauer. *The $L^p$ conjecture for a finitely additive measure*

Let $s = \{s_n\}$ be a sequence and let a point $\rho_0$ in $I$ be fixed, where $I$ is the unit interval $[0, 1)$ and the symbol $X$ denotes the Stone-Cech