HIGHER ORDER COMPOSITIONS IN THE ADAMS SPECTRAL SEQUENCE

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Communicated by P. Emery Thomas, December 3, 1969

Following is a summary of results relating to the convergence of Massey products in the Adams spectral sequence to Toda brackets in stable homotopy. These results should give much useful information on the classical structure problems; detecting infinite cycles, computing differentials and describing the extensions in E_∞.

Forthcoming papers will treat Proposition II and Theorems III, IV, and V in full. R. M. F. Moss [6] has already proven Theorem III in the case of secondary compositions.

1. The Adams spectral sequence. We presuppose an appropriate stable category $\mathcal{S}_*$ such as Boardman's C-W spectra [2]. Let X, Y be objects of $\mathcal{S}_*$. We denote Hom $_{\mathcal{S}_*}(X, Y)$ by $[X, Y]_*$. As in Adams [1], we may construct a spectral sequence $\{E_r^{**}(X, Y)\}$ for which

$$E_2^{**}(X, Y) = \mathrm{Ext}_{A(2)}^{**}(H^*(Y; Z_2), H^*(X; Z_2))$$

and $E_\infty^{**}(X, Y)$ is induced by a filtration on the 2-primary part of $[X, Y]_*$.

The main technical results which enable us to obtain convergence of Massey products to Toda brackets are Propositions I and II below.

PROPOSITION I (R. M. F. MOSS). *Let X, Y, Z, W be objects of $\mathcal{S}_*$. For $2 \leqq r$ there are associative pairings*

$$E_r^{s,t}(Y, X) \otimes E_r^{s',t'}(W, Y) \to E_r^{s+s',t+t'}(W, X)$$

and these pairings have the following properties:

(i) *Let $\alpha \in E_r^{s,t}(Y, X)$, $\beta \in E_r^{s',t'}(W, Y)$, $d_r''(\alpha\beta) = d_r(a)\beta + ad_r'(\beta)$ where d_r, d_r', d_r'' are the differentials in the appropriate Adams spectral sequences.*

(ii) *The isomorphisms $E_{r+1} \cong H(E_r)$ commute with the pairings.*

(iii) *The composition pairings $[Y, X]_p \otimes [W, Y]_q \to [W, X]_{p+q}$ preserve filtration; on passing to quotients and using the isomorphisms of* (ii) *the composition pairing of the E_∞ terms is obtained.*

AMS Subject Classifications. Primary 5545, 5552.

Key Words and Phrases. Adams spectral sequence, Massey product, Toda bracket, splitting of Adams spectral sequence, convergence of Massey products in the Adams spectral sequence, generalized Leibnitz formulae.