BRANCHING PROCESSES WITH RANDOM ENVIRONMENTS[1]

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Communicated by David Blackwell, December 4, 1969

Let (Ω, F, P) be a probability space. Let $\mathfrak{M}$ designate the collection of all probability distributions $\{\bar{p}=\{p_i\}_{i=0}^{\infty}, p_i \geqq 0, \sum p_i = 1\}$ on the nonnegative integers satisfying the further constraints

$$\sum_{i=0}^{\infty} i p_i < \infty, \qquad 0 \leqq p_0 + p_1 < 1. \tag{1}$$

Let $\zeta_i(\omega)$; $i=0, 1, 2, \cdots$ be a sequence of random mappings from (Ω, F, P) into $(l_\infty, \mathcal{B}_\infty)$ where $\mathcal{B}_\infty$ is the Borel σ-algebra in l_∞ (the Banach space of bounded sequences of real numbers) generated by the product topology. We assume

$$P\{\omega; \zeta_i(\omega) \in \mathfrak{M} \quad \text{for all } i\} = 1. \tag{2}$$

For any $\zeta \in \mathfrak{M}$ associate the p.g.f.

$$\phi_\zeta(s) = \sum_{i=0}^{\infty} p_i(\zeta) s^i \qquad |s| \leqq 1. \tag{3}$$

Let $Z_n(\omega), n \geqq 0$ be a sequence of nonnegative integer valued random variables defined on (Ω, F, P). For any collection D of random elements on (Ω, F, P) let $\sigma(D)$ denote the sub σ-algebra of F generated by D. Set

$$\begin{aligned} F_n(\bar{\zeta}) &= \sigma(\zeta_0, \zeta_1, \cdots, \zeta_n), \qquad F(\zeta) = \bigcup_{n \geqq 1} F_n(\bar{\zeta}), \\ F_{n,z}(\bar{\zeta}) &= \sigma(\zeta_0, \zeta_1, \cdots, \zeta_n, Z_0, Z_1, \cdots, Z_n). \end{aligned} \tag{4}$$

We postulate that $\{Z_n\}$ satisfy the recurrence relations

$$E(s^{Z_{n+1}} \mid F_{n,z}) = [\phi_{\zeta_n}(s)]^{Z_n} \qquad \text{a.s.} \tag{5}$$

and for any set of integers $1 \leqq n_1 < n_2 < \cdots < n_k$ with $|s_i| \leqq 1$,

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AMS Subject Classifications. Primary 6067; Secondary 6030.

Key Words and Phrases. Branching processes, random environments, stationary ergodic process, extinction probability, limit theorems.

[1] Sponsored in part by the Mathematics Research Center, United States Army, under contract No. DA-31-124-ARO-462, University of Wisconsin, Madison, Wisconsin, 53706, and in part under contract NOO14-67-A-0112-0015 at Stanford University.