HÖLDER AND L^p ESTIMATES FOR SOLUTIONS OF $\bar{\partial} u=f$ IN STRONGLY PSEUDOCONVEX DOMAINS[1]

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1. **Introduction.** A recent result due to H. Grauert and I. Lieb [1] asserts that if G is a strongly pseudoconvex domain with smooth boundary, $G \subseteq C^n$, and if $f=\sum_{j=1}^n f_j\, d\bar{z}_j$ is a C^∞, (0, 1) form in G, $\bar{\partial} f=0$, f bounded, then the equation $\bar{\partial} u=f$ has a solution $u: G \rightarrow C$ such that $\sup_{z \in G}|u(z)| \leqq C \sup_{z \in G}|f(z)|$, where $|f(z)|=\sum_{j=1}^n|f_j(z)|$. Grauert and Lieb's theorem is proved by writing a solution u in the form $u(w)=\int_G \Omega(z, w) \wedge f(z)$, $w \in G$ and then estimating the kernel to obtain $\int_G|\Omega(z, w)| dz \leqq A<\infty$, A independent of $w \in G$. The kernel $\Omega(z, w)$ is the one constructed by E. Ramirez in [6] who employed it to obtain an integral representation formula for holomorphic functions. Ramirez' construction of $\Omega(z, w)$ involves the application of Cartan's theorem B for vector valued functions as well as a division theorem which he proves in [6].

We have found an alternate approach using Hörmander's L^2 estimates which yields a somewhat simpler proof: We first determine (Theorem L) a local solution by the same method as in Grauert and Lieb's paper. In this local case, however, a kernel $\Omega(z, w)$ can be written explicitly. Our passage from local to global then uses only Hörmander's L^2 estimates for the $\bar{\partial}$ problem. By this method we obtain a stronger result, namely a solution u satisfying a Hölder condition with any exponent α, $\alpha<1/2$, up to the boundary of G (Theorem 1). The method also yields (Theorem 2) solutions in L^p whenever $f \in L^p$, $1 \leqq p \leqq \infty$; this is not an interpolation result even for $2 \leqq p \leqq \infty$ (see remarks following Theorem 2).

As an application of Grauert-Lieb's theorem we prove (Theorem 3) that holomorphic functions which are continuous up to the boundary of G can be uniformly approximated on G by holomorphic functions defined in a neighborhood of $\bar{G}$. This result has been proved independently and at about the same time by I. Lieb [5] using the Ramirez integral formula.

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[1] The results of this note are part of the author's thesis at New York University, Courant Institute of Mathematical Sciences. The proofs will appear in full elsewhere.