TWO NEW H-SPACES

BY MORTON CURTIS[1] AND GUIDO MISLIN[2]

Communicated November 24, 1969

It is the purpose of this note to announce the following result.

THEOREM. (i) *The total space of any principal* $SU(3)$ *bundle over* S^7 *is an H-space.*

(ii) *There are exactly four homotopy types of such total spaces.*

Two of these homotopy types are known H-spaces; namely, $SU(3)\times S^7$ and $SU(4)$. The other two are the new H-spaces of the title, and a word is in order as to in what sense they are new.

If one seeks differentiable manifolds which are H-spaces not homeomorphic to known H-spaces, then recent work of Belfi [1] and Morgan [4] furnish a big supply. For example, there are infinitely many nonhomeomorphic manifolds having the homotopy type of $SU(4)$ (and hence being H-spaces). If one seeks new homotopy types (excluding, of course, cartesian products of known ones) the picture is quite different. Classically one knew only S^7 and its projective space P^7, except for Lie groups. In 1968 Hilton and Roitberg [2], [3] discovered a new H-space, a principal S^3 bundle over S^7. In 1969 Stasheff [5] found two more new H-spaces among the seven homotopy types of principal S^3 bundles over S^7. Our two new spaces brings the known total to seven in dimension $\leqq 15$.[3] We have also shown that the three new homotopy types introduced by going from principal S^3 bundles over S^7 to $SO(4)$ 3-sphere bundles over S^7 are not H-spaces.

The first part of our theorem is proved using the technique of mixing homotopy types (relative to a subdivision of the set of prime numbers) due to Zabrodsky [7], in much the same manner as Stasheff [5]. The second part uses the Adams operations in K-theory and a result of Suter [6] to distinguish the homotopy types.

REFERENCES

1. Victor Belfi, *Nontangential homotopy equivalences*, Notices Amer. Math. Soc. 16 (1969), 585. Abstract #69T-G51.

AMS Subject Classifications. Primary 1982, 1970.

Key Words and Phrases. H-space, homotopy type.

[1] Research partially supported by NSF Grant No. GP12715.

[2] Research supported by the Schweizerischer Nationalfonds.

[3] By using [7] and a theorem of W. Browder, Zabrodsky gets infinitely many H-manifolds in higher dimensions. Recently Roitberg has used [7] to obtain some new 14-dimensional H-manifolds.