THE CONDITION NUMBER OF A CLASS OF RAYLEIGH-RITZ-GALERKIN MATRICES[1]

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The purpose of this note is to study the Euclidean condition number of the matrix resulting from using the well-known Rayleigh-Ritz-Galerkin method with finite dimensional subspaces of polynomial spline functions to approximate the solution of a linear, self-adjoint, two-point boundary value problem. Roughly speaking, we consider a model class of such problems of order $2n$ and determine *upper bounds*, of the form of a constant times the norm of the partition associated with the spline subspace to the $-2n$th power, for the Euclidean condition number of the associated matrix.[2]

The class of problems we are considering is defined by

$$(1)\quad L[u] \equiv \sum_{j=0}^{n} (-1)^j D^j[p_j(x) D^j u(x)] = f(x), \qquad -\infty < x < \infty,$$

subject to the boundary conditions

$$(2)\qquad \lim_{x\to\infty} D^k u(x) = \lim_{x\to-\infty} D^k u(x) = 0, \qquad 0 \leqq k \leqq n-1,$$

where $D \equiv d/dx$.

Let $H_0^{n,2}$ be the completion of the real-valued functions, $h(x)$, in $C_0^\infty(-\infty, \infty)$, i.e., the completion of the infinitely differentiable, real-valued functions with compact support, with respect to the Sobolev norm

$$\|h\|_n \equiv \left(\int_{-\infty}^{\infty} [D^n h(x)]^2 dx\right)^{1/2}.$$

Moreover, we assume that $p_j(x) \in L^\infty(-\infty, \infty)$ and are real-valued for $0 \leqq j \leqq n$, $f(x) \in L^2(-\infty, \infty)$ and is real-valued, and that

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[2] After sending this manuscript to the editor we learned that G. Fix and G. Strang have obtained analogous results for the case of uniform partitions by means of Fourier transform techniques.