A $P(\phi)$ QUANTUM FIELD THEORY[1]

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1. **Introduction.** A long-standing problem of quantum field theory is to prove the existence of solutions to the field equations for realistic physical models. The model we consider is that of a self-interacting boson field in two-dimensional space-time with self-interaction given by an arbitrary semibounded polynomial in the field. The hamiltonian supplied by the physics is given formally in terms of the field ϕ as the sum of a free term and an interaction term:

$$H = H_O + H_I \equiv \frac{1}{2}\int :(\phi_x^2 + \phi_t^2 + m^2\phi^2):dx + \int :P(\phi(x)):dx.$$

Here $m>0$ is the bare mass of the boson, $P(y) = b_{2n}y^{2n} + b_{2n-1}y^{2n-1} + \cdots + b_0$ is a polynomial with $b_{2n}>0$, and the colons represent the operation of Wick or normal ordering (defined below).

The corresponding classical field equation is

$$\phi_{tt} - \phi_{xx} + m^2\phi + P'(\phi) = 0 \tag{1}$$

where (classically) ϕ is a real-valued function of x and t. In quantum field theory, ϕ is a distribution in x and t whose values are operators in some Hilbert space; we seek such a solution ϕ of (1).

The natural Hilbert space for noninteracting bosons is (momentum) Fock space, $\mathfrak{F} = \sum_n \oplus \mathfrak{F}^n$. Here $\mathfrak{F}^0 = \boldsymbol{C}$, $\mathfrak{F}^1 = L_2(\boldsymbol{R})$, and $\mathfrak{F}^n$ is the n-fold symmetric tensor product of $\mathfrak{F}^1$. Thus a vector $\Psi \in \mathfrak{F}$ is a sequence of n-particle vectors $\Psi = (\Psi_0, \Psi_1, \cdots)$ where $\Psi_n(p_1, \cdots, p_n)$ is a symmetric function of n momentum variables. The annihilation operator $a(k)$ on $\mathfrak{F}$ maps $\mathfrak{F}^n$ into $\mathfrak{F}^{n-1}$:

$$(a(k)\Psi)_{n-1}(p_1, p_2, \cdots, p_{n-1}) = n^{1/2}\Psi_n(p_1, \cdots, p_{n-1}, k).$$

The formal adjoint of $a(k)$ is

$$(a^*(k)\Psi)_{n+1}(p_1, p_2, \cdots, p_{n+1}) = (n+1)^{1/2}S\delta(p_{n+1} - k)\Psi_n(p_1, \cdots, p_n)$$

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