GENERALISED NUCLEAR MAPS IN NORMED LINEAR SPACES

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1. **Preliminary definitions and notations.** Grothendieck [3] and Pietsch [6] present an exhaustive study of nuclear operators and nuclear maps. The notion of a nuclear operator was extended by Persson and Pietsch in a recent paper [5] and they study in detail the p-nuclear and quasi-p-nuclear maps. In this paper we define and study certain linear maps called λ-nuclear and quasi-λ-nuclear maps. Our definition and generalisation here are motivated by the Köthe sequence spaces and their duality theory. For the special case $\lambda = l^1$ we obtain the nuclear operators and for $\lambda = l^p$ we obtain the p-nuclear maps; also, the special case $\lambda = c_0$ yields the ∞-nuclear operators of Persson and Pietsch. Most of the results in this work are motivated by the work of Persson and Pietsch [5] and Köthe sequence spaces.

We shall briefly outline our assumptions. For definitions not stated here see Garling [1], Köthe [4], Ruckle [7], Sargent [9] and Zeller [10]. Let λ be a symmetric sequence space of scalars and λ^* be its Köthe dual. We shall assume that λ is provided with the Mackey topology of the duality $\langle \lambda, \lambda^* \rangle$ and that this topology is provided by a norm p, p itself being an extended seminorm on ω. We assume now that λ is solid and that it is K-symmetric, i.e., for each $x \in \lambda$ and for each permutation π of I^+ we have $x_\pi \in \lambda$ and $p(x) = p(x_\pi)$. λ is also assumed to be a BK space with AK. We remark that our assumptions imply that $\lambda = \omega$ or $\lambda = l^\infty$ or $\lambda \subseteq c_0$. The space λ^* is now considered as the topological dual of λ and equipped with its natural norm topology.

We pause now to point out that in addition to the spaces l^p, $1 \leq p < \infty$, the sequence spaces $n(\phi)$ of Sargent [8] and the sequence spaces $\mu_{a,p}$ and $\nu_{a,p}$ of Garling [2] serve as examples of the type of sequence spaces λ we consider. Garling shows also that his spaces $\mu_{a,p}$ are in general not linearly homeomorphic to l^p.

Next let E and F be normed linear spaces. Then $\lambda(E)$ is the (vector sequence) space of all vectors $x = (x_n)$, $x_n \in E$ for each n and such that the sequence $(\langle x_n, a \rangle) \in \lambda$ for each $a \in E'$. Formally define

$$\epsilon_\lambda(x) = \sup_{\|a\| \leq 1} p(|\langle x_n, a \rangle|),$$

where p is the norm on λ.

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