ON THE COHOMOLOGY OF STABLE TWO STAGE POSTNIKOV SYSTEMS

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Communicated by Norman Steenrod, February 2, 1970

Introduction. Let $\xi=(E, p, B, F)$ denote a two stage Postnikov system with stable k-invariant. We announce results about $H^*(\Omega E)$ as a Hopf algebra over the Steenrod algebra. Mod 2 cohomology is used exclusively. Unexplained notation is from [4] and [5]. I am grateful to D. Anderson, W. Massey, F. Peterson and H. Salomonsen for many useful remarks.

We make the following assumptions on ξ, in addition to those of [5, p. 38]. F and B are simply connected products of finitely many Eilenberg-MacLane spaces. The nonzero homotopy groups of the factors of B are infinite cyclic or cyclic of order 2^k, $k=1, 2, \cdots$. All factors of F have Z_2 (cyclic group of order 2) as their only nonzero homotopy group.

Results of [3], [4], [5] and [8] give $H^*(\Omega E)\cong R\otimes U(X')$. The isomorphism is as algebras over Z_2 and $\otimes$ is over Z_2. $R=R(\Omega\xi)=H^*(\Omega B)/\ker \Omega p^*$ and $X'=X'(\Omega\xi)$ is considered as known, [5, p. 54]. In general $H^*(\Omega E)$ does not split this way as a Hopf algebra over Z_2. The new result is Theorem A. It gives $H^*(\Omega E)$ as a coalgebra over R. It also gives information on the extension problem represented by the fundamental sequence of $\Omega\xi$ [5, p. 54]. This use of the Hopf algebra structure is well known, [1], [5] and [6].

1. **The main theorem.** Consider the following diagram of unstable A-modules and A-maps. The squares are commutative.

$$X'(\Omega\xi) \xleftarrow{\alpha} Y''/\lambda Y'' \xleftarrow{\pi} Y'' \xrightarrow{c} \Omega Y, \tag{1}$$

$$\begin{array}{ccccc} Y & \xrightarrow{f^*} & Z & \xrightarrow{\rho} & Z' \\ \sigma_{B_0}\downarrow & & \sigma_B\downarrow & & \sigma'\downarrow \\ \Omega Y & \xrightarrow[\Omega f^*]{} & \Omega Z & \xrightarrow[\rho']{} & \Omega Z' \end{array} . \tag{2}$$

Here α is an A-isomorphism of degree -1; π, ρ and ρ' are natural projections; c is inclusion, and σ' is the obvious map. The remaining

AMS Subject Classifications. Primary 5550; Secondary 5534.

Key Words and Phrases. Postnikov system, stable k-invariant, Steenrod algebra, Hopf algebra, Massey-Peterson fundamental sequence.