HOMOGENEOUS POSITIVELY PINCHED RIEMANNIAN MANIFOLDS

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1. **Introduction.** The purpose of this paper is to announce several results on homogeneous positively pinched manifolds. We give, modulo two examples, a complete classification of even dimensional homogeneous spaces that admit positively pinched homogeneous Riemannian structures (see §5, Theorem 5.2). These spaces are all diffeomorphic with rank one symmetric space except for two possible exceptions $SU(3)/T$ (T the maximal torus in $SU(3)$) and

$$Sp(3)/SU(2) \times SU(2) \times SU(2).$$

2. **Preliminary results.** Let G be a compact connected Lie group and let K be a closed subgroup of G. Let $M = G/K$ and suppose that M carries a G-invariant positively pinched Riemannian structure (that is all sectional curvatures are bounded below by a positive constant).

PROPOSITION 2.1. *If M is even dimensional then G is semisimple. If M is odd dimensional then G is either semisimple or G has a one-dimensional center.*

This result is proved by analyzing directly the curvature of M as a 4-linear form on a suitably chosen $\mathrm{Ad}(K)$-invariant complement $\mathfrak{p}$ to the Lie algebra of K, in the Lie algebra of G, $\mathfrak{g}$ (in particular $\mathfrak{p}$ is chosen so that it contains the center of $\mathfrak{g}$). The even dimensional statement will be considerably strengthened in the next section.

The next result indicates the difficulties in using a Lie theoretic, algebraic approach to the general study of homogeneous positively pinched Riemannian manifolds.

PROPOSITION 2.2. *Let (G, K) be as in Proposition* 1. *Let T be a maximal torus in K. Let $C(T)$ be the centralizer of T in G. Let $p_0 = eK$ (the identity coset in M) and let $M_T = C(T)p_0$. Then*

(1) *M_T is totally geodesic in M.*

(2) *$C(T)/T$ is either one dimensional or semisimple.*

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