GROMOLL GROUPS, Diff S^n AND BILINEAR CONSTRUCTIONS OF EXOTIC SPHERES

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Communicated by William Browder, February 20, 1970

1. Introduction and main results. The Kervaire-Milnor group Γ^n has a filtration by subgroups,

$$0 = \Gamma^n_{n-1} \subset \cdots \subset \Gamma^n_{k+1} \subset \Gamma^n_k \subset \cdots \subset \Gamma^n_1 = \Gamma^n,$$

due to Gromoll [9], which we study by means of certain homomorphisms

$$\begin{tikzcd}[column sep=small] \pi_p(SO_q) \otimes \pi_q(SO_p) \arrow[dr, "\sigma_{p,q}"'] & & \Gamma^{p+1} \otimes \pi_q(SO_p) \arrow[dl, "\tau_{p+1,q}"] \\ & \Gamma^{p+q+1} & \end{tikzcd}$$

See [12] for definitions. The pairing σ was first introduced by Milnor [13] and has been studied in [3], [11]. The pairing τ has been studied in [8], [16].

The groups of Gromoll are related to the homotopy groups of Diff S^n by a simple pasting construction: namely, there are homomorphisms $\lambda_i:\pi_i(\text{Diff } S^n)\rightarrow\Gamma^{n+i+1}$ with image $\lambda_i=\Gamma^{n+i+1}_{i+1}$ (see Proposition 2.1 and also [9, §1]).

We shall detect nontrivial elements in some Γ^n_{k+1}. Note that $\Gamma^n_{k+1}\neq 0$ implies that $\Gamma^n_{i+1}\neq 0$ and, hence, $\pi_i(\text{Diff } S^{n-i-1})\neq 0$, for all $i\leqq k$. For slightly sharper statements see Proposition 3.3 and Proposition 3.4.

1.1. THEOREM. (a) $\Gamma^{4k-1}_{2k-2}\neq 0$, *for all* $k\geqq 4$.
(b) $\Gamma^{4k+1}_{2v(k)}\neq 0$, *for all* $k\geqq 0$, $k\neq 2^l-1$.

Here $v(k)$ is the maximum number of linearly independent vector fields on S^{2k+1}. It is well known that $v(k)=1$ when k is even and $v(k)\geqq 3$, when k is odd. Its precise value is given in [2].

Theorem 1.1 follows from some of our results on σ. Corollary 3.5, below, also based on work with σ, actually establishes fairly large lower bounds for the order of Γ^{4k-1}_{2k-2} (with some restrictions on k).

AMS Subject Classifications. Primary 5710, 5755; Secondary 5322.

Key Words and Phrases. Kervaire-Milnor group of exotic spheres, Γ^n, Gromoll filtration of Γ^n, group of self-diffeomorphisms, homotopy type of CW complex, homotopy-abelian H-space, inertia groups of manifolds, sectional curvature, pinching, bilinear pairings of Milnor-Munkres-Novikov.

[1] Work of all authors supported in part by National Science Foundation.