THE SINGULAR SETS OF AREA MINIMIZING RECTIFIABLE CURRENTS WITH CODIMENSION ONE AND OF AREA MINIMIZING FLAT CHAINS MODULO TWO WITH ARBITRARY CODIMENSION[1]

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1. When describing the interior structure of an area minimizing m dimensional locally rectifiable current T in R^{m+1}, one calls a point $x \in \operatorname{spt} T \sim \operatorname{spt} \partial T$ regular or singular according to whether or not x has a neighborhood V such that $V \cap \operatorname{spt} T$ is a smooth m dimensional submanifold of R^{m+1}. As a result of the efforts of many geometers it is known that there exist no singular points in case $m \leqq 6$; a detailed exposition of this theory may be found in [3, Chapter 5]. Recently it was proved in [2] that

$$Z = \partial(E^8 \llcorner R^8 \cap \{x : x_1^2 + x_2^2 + x_3^2 + x_4^2 < x_5^2 + x_6^2 + x_7^2 + x_8^2\})$$

is a 7 dimensional area minimizing current in R^8 with the singular point 0. This implies that, for $m > 7$, $E^{m-7} \times Z$ is an m dimensional area minimizing current in $R^{m-7} \times R^8 \simeq R^{m+1}$ with the $m-7$ dimensional singular set $R^{m-7} \times \{0\}$. Here we will show (Theorem 1) that the Hausdorff dimension of the singular set of an m dimensional area minimizing rectifiable current in R^{m+1} never exceeds $m-7$.

Our method also yields the result (Theorem 2) that the Hausdorff dimension of the singular set of an m dimensional area minimizing flat chain modulo 2 in R^{m+p} never exceeds $m-2$, for arbitrary codimension p.

2. We use the terminology of [3]. Given any positive integer m we choose Υ according to [3, 5.4.7] with $n = m+1$ and let

$$\omega(T) = \{x : \Theta^m(\|T\|, x) \geqq \Upsilon\} \quad \text{for } T \in \mathcal{R}_m^{\mathrm{loc}}(R^{m+1}).$$

Whenever $0 \leqq k \in R$ and $A \subset R^{m+1}$ we define $\phi_\infty^k(A)$ as the infimum of the set of numbers $\sum_{B \in G} \alpha(k) 2^{-k} (\operatorname{diam} B)^k$ corresponding to all countable open coverings G of A. We see from [3, 2.10.2] that

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