ON THE VIETORIS-BEGLE THEOREM[1]

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The usual forms of the Vietoris-Begle theorem require vanishing of the first p-cohomology groups of the antecedents of points under the mapping. The present work utilizes a multi-cone construction to yield a variety of results when either one cohomology group alone vanishes or when several cohomology groups, not necessarily including that of dimension 0, vanish. Some converse theorems are obtained relating the global conditions on the cohomology groups of the space and the vanishing of some of the cohomology groups of point antecedents. These results demand special conditions on the space and its map and trivial examples show that otherwise the theorems obtained are invalid. For the metric case Borsuk [1] used a somewhat different cone construction. His space and map restrictions are much stronger than those introduced here.

The cohomology groups are understood to be the reduced groups. The coefficient group is a fixed Abelian group. Throughout let $f: X \to Y$ be surjective and continuous and X and Y are compact Hausdorff. Denote by X_y the set $\{x \mid f(x) = y\}$. Write I_y for the segment $[01]$ over y and Π_{Y0} for the product $\prod_{y \in Y_0} I_y$, $Y_0 \subset Y$ in the usual Tychonoff topology. If $Y_0 = Y$ write Π. Thus the elements of Π_{Y0} are functions, ψ, on Y_0 to I. Designate by 1_{y_0} the function for which $1_{y_0}(y) = 0$, $y \neq y_0$ and $1_{y_0}(y_0) = 1$ and let $J_{y_0} = \{\psi \mid \psi(y) = 0,\ y \neq y_0,\ \psi(y_0) \in I_y\}$. Denote by $*$ the identically 0 function. Let $P(Y_0) = X \times \Pi(Y_0)$ with the product topology and write $B(y_0) = X_y = J_y \subset P$. The cylinder set B is $\bigcup_{y \in Y_0} B_y \subset P$. Intuitively B consists of cylinders sticking out in different directions.

It is easy to show that B is closed in the compact space P and so is compact. It is evidently Hausdorff also. Now collapse the roofs of the cylinders, B_y, in B to yield cones. More precisely identify $X_y \times 1_y$ to a peak point denoted by x_y^*. The resulting cone is denoted by X_y^* and the collection of cones by $'X$ where we assume the identification or quotient topology. Then

LEMMA 1. *$'X$ is compact.*

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