ON STRUCTURAL STABILITY[1]

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The purpose of this note is to sketch a proof of

THEOREM A. *A C^2 diffeomorphism (on a compact, boundaryless manifold) which satisfies Axiom* A *and the strong transversality condition is structurally stable.*

This is (one direction of) a conjecture of Smale [3]. The case where the nonwandering set is finite is the main theorem of [4]. For background, see [2] and [3]. Details will be given in a subsequent publication.

1. **An infinitesimal condition.** Throughout, M denotes a smooth, compact, boundaryless manifold and $f: M \to M$ a diffeomorphism. A *chart* on M is a pair (α, U) where U is an open subset of M and α maps a neighborhood of $\bar{U}$ diffeomorphically onto an open subset of Euclidean space $\mathbf{R}^m$.

Let $\mathfrak{X}^0(M)$ denote the Banachable space of all continuous vector fields on M. Let $f^{\#}: \mathfrak{X}^0(M) \to \mathfrak{X}^0(M)$ be the continuous linear operator defined by $f^{\#}\eta = Tf^{-1} \circ \eta \circ f$ for $\eta \in \mathfrak{X}^0(M)$.

Fix a Riemannian metric on M and let d denote the corresponding metric on M; i.e., for $x, y \in M$, $d(x, y)$ is the infimum of the lengths of all curves from x to y. We define a new metric d_f by

$$d_f(x, y) = \sup_n d(f^n(x), f^n(y))$$

where the supremum is over all integers n. Let $\mathfrak{X}_f(M)$ denote the set of all $\eta \in \mathfrak{X}^0(M)$ with the property that for every chart (α, U) on M there exists $K > 0$ such that

$$| \eta_\alpha(x) - \eta_\alpha(y) | \leqq K d_f(x, y)$$

for all $x, y \in U$. Here $\eta_\alpha: U \to \mathbf{R}^m$ is defined by $T\alpha \circ \eta(x) = (\alpha(x), \eta_\alpha(x))$ for $x \in U$. By standard techniques $\mathfrak{X}_f(M)$ can be made into a Banachable space. The inclusion $\mathfrak{X}_f(M) \to \mathfrak{X}^0(M)$ is continuous and for any finite cover of M by charts (α, U) the K's above can be chosen small if η is sufficiently close to 0 in $\mathfrak{X}_f(M)$.

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