THE SECOND FUNDAMENTAL THEOREM FOR MEROMORPHIC MINIMAL SURFACES[1]

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ABSTRACT. The second fundamental theorem of Nevanlinna concerning meromorphic functions of a complex variable is extended in this note to an analogous result for meromorphic minimal surfaces. A similar extension of the first fundamental theorem involved generalizations of the classical proximity and enumerative functions and also a new visibility function; for the present result, a second enumerative function and a second visibility function are defined. Defect relations are discussed.

1. **Introduction.** The second fundamental theorem of Nevanlinna concerning meromorphic functions of a complex variable [4, p. 227] is extended in this note to an analogous result for meromorphic minimal surfaces. A full presentation of the material here briefly outlined will appear elsewhere. Details of the proof of a similar extension of the first fundamental theorem can be found in [1].

Let a surface

$$S: x_j = x_j(u, v), \qquad j = 1, 2, 3, \tag{1}$$

be given in isothermal representation, that is, in a representation for which

$$E(u, v) = G(u, v), \qquad F(u, v) = 0, \tag{2}$$

where E, F, and G are the coefficients of the first fundamental quadratic form of S. Then S is a minimal surface if and only if the coordinate functions (1) are harmonic.

If S is a minimal surface in isothermal representation, then the poles and the finite $\boldsymbol{a}$-points, where $\boldsymbol{a} = (a_1, a_2, a_3)$, of S are isolated, as are the infinities and the zeros of the area-deformation function E [1].

A *meromorphic minimal surface* is a minimal surface, with harmonic coordinate functions (1) satisfying (2), which has no singularities other than poles for $u^2 + v^2 < \infty$.

2. **The first fundamental theorem.** In the stereographic projection of extended Euclidean 3-space onto the hypersphere

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