AN INVARIANCE PRINCIPLE FOR THE EMPIRICAL PROCESS WITH RANDOM SAMPLE SIZE

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Communicated by David Blackwell, January 19, 1970

Let $C=C[0, 1]$ be the space of continuous functions on $[0, 1]$ with the uniform topology, that is the distance between two points x and y (two functions x and y of $t\in[0, 1]$) is defined by

$$\rho(x, y) = \sup_t |x(t) - y(t)| .$$

Let $\mathcal{B}$ be the σ-field of Borel sets of C.

Let $(\Omega, \mathcal{A}, P)$ be some probability space and W be the Wiener measure on $(C, \mathcal{B})$ with the corresponding Wiener process $\{W_t(\omega):0\leqq t\leqq 1\}$, $\omega\in\Omega$; that is W_t has values in C and is specified by $E(W_t)=0$ and $E(W_sW_t)=s$ if $s\leqq t$. Let W^0 be the Gaussian measure on $(C, \mathcal{B})$ constructed by setting $W_t^0=W_t-tW_1$. Then $W_t^0\in C$, $E(W_t^0)=0$ and $E(W_s^0W_t^0)=s(1-t)$ if $s\leqq t$. Also $W_0^0=W_1^0=0$ with probability 1 and $\{W_t^0:0\leqq t\leqq 1\}$ is called the tied down Wiener process or the Brownian bridge.

Let $S_n=\xi_1+\cdots+\xi_n$, $S_0=0$, $n=1, 2, \cdots$ be the partial sum sequence of random variables $\{\xi_n\}$ defined on $(\Omega, \mathcal{A}, P)$. Define a random element X_n of C by

$$X_n(t, \omega) = W_n(t, \omega) + (nt - [nt])\xi_{[nt]+1}(\omega)/n^{1/2} - tW_n(1, \omega) \tag{1}$$

where $W_n(t, \omega)=S_{[nt]}(\omega)/n^{1/2}$. The following theorem is an immediate consequence of L. Breiman's analysis of §§13.5 and 13.6 in his book [3].

THEOREM B. *Suppose the random variables $\xi_1, \xi_2, \cdots$ are independent and identically distributed with mean zero and variance* 1. *Then the random functions X_n defined by* (1) *satisfy*

$$X_n \xrightarrow{\mathcal{D}} W^0. \tag{2}$$

Here (2), and also similar relations later on, are interpreted in accordance with (4.5) and (4.7) of Billingsley's book [2], depending on

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AMS Subject Classifications. Primary 6030, 6040; Secondary 6270, 6271.

Key Words and Phrases. Invariance principle, weak convergence, randomly selected partial sums, empirical process, random-sample-size Kolmogorov-Smirnov statistics.

[1] Work done while the author was a Canada Council Fellow in 1969–1970 at the University of Vienna.