ENERGY FLOW: WAVE MOTION AND GEOMETRICAL OPTICS[1]

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ABSTRACT. Energy distribution for solutions of the wave equation in the presence of a reflecting body can be investigated with varying degrees of refinement by using quadratic inequalities, Huyghens principle and geometrical optics. The relations between these properties and their validity in general cases is discussed and some of the simpler proofs outlined.

1. Introduction. This paper deals with the motion of *conservative* systems, i.e. systems whose total energy remains the same for all time. Energy in general is a positive functional of the instantaneous state of the system; for systems governed by linear equations energy is a *positive definite quadratic form*. Conservation of energy implies that if the initial state is zero, so are all subsequent and previous states; for linear systems we conclude that solutions are uniquely determined by their initial data. For solutions with nonzero initial states the conservation of energy furnishes an a priori bound which, when combined with orthogonal projection techniques, yields a proof of the existence of solutions with arbitrarily prescribed initial data with finite energy.

Thus for conservative systems we can prove rather easily the existence and uniqueness of solutions. Current research is directed at establishing more detailed properties of the way solutions evolve in time, in particular for systems located in an unbounded domain interest focuses on the following problems:

I. *Energy flow.* In the course of time where will the energy or amplitude of a solution be concentrated? Will it eventually diffuse to infinity?

II. *Transport of singularities.* If the data contain singularities how and where are they propagated?

III. *Asymptotic description.* What is the behavior of highly oscillatory solutions? Can they be described by a simplified theory? An

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