ON THE COMPLEX BORDISM AND COBORDISM OF INFINITE COMPLEXES

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Let $MU_*(\ )$ and $MU^*(\ )$ denote the reduced homology (complex bordism) and cohomology (complex cobordism) functors represented by the unitary Thom spectrum MU. Two examples of the immense richness of these functors are provided by the development (for suitable complexes or spectra) of the Adams spectral sequence

$$(1)\qquad \mathrm{Ext}^{*,*}_{MU^*(MU)}(MU^*(X), MU^*(Y)) \Rightarrow \{Y, X\}_*$$

by S. P. Novikov [12], and the universal coefficient theorem

$$(2)\qquad \mathrm{Tor}^{MU_*(S^0)}_{*,*}(MU_*(X), \boldsymbol{Z}) \Rightarrow H_*(X; \boldsymbol{Z})$$

by P. E. Conner and L. Smith [9]. Recently N. A. Baas [4] has written an excellent account of the Adams spectral sequence (1), and J. F. Adams has made a thorough analysis of universal coefficient theorems such as (2) in Lecture 1 of [1].

In §1 we announce several solutions to the problem—when is $MU^*(X)$ isomorphic to the inverse limit of the complex cobordism of the skeleta (assumed finite) of X? In the remaining sections we illustrate several universal coefficient theorems, among them (2), by announcing the results of several computations for Eilenberg-MacLane spectra $K(\pi)$ and the spectrum bu which represents connective K-theory. Full details will appear elsewhere.

1. Let X denote a based CW-complex or highly connected CW-spectrum as defined by J. M. Boardman [5]. We shall assume that each skeleton X^μ of X is a finite complex, and define a filtration of $MU^t(X)$ by the subgroups $MU^t_\mu(X) = \mathrm{Ker}\{MU^t(X) \to MU^t(X^{\mu-1})\}$. In [4] Baas emphasizes the importance, for the construction of the Adams spectral sequence (1), of dealing with spectra for which the filtration topology on the cobordism groups $MU^t(X)$ is complete and Hausdorff. In [6] and [7] V. M. Buhštaber and A. S. Miščenko

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