REPRESENTATIONS OF INFINITE DIMENSIONAL MANIFOLDS AND $\infty - p$ HOMOLOGY FUNCTORS

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Introduction. The purpose of this note is to announce a representation theorem for separable Fréchet manifolds. This representation theorem demonstrates a close connection between functionals and infinite dimensional spaces. Moreover, it can be applied for the canonical construction of an $\infty - p$ homology functor.

1. **The representation theorem.** Throughout this note $\approx$ will denote homeomorphism or isomorphism, $\sim$ a diffeomorphism, and $\simeq$ a strong homotopy equivalence. Also manifolds are connected.

THEOREM A. *Let E be a separable C^∞ manifold without boundary modeled on the Hilbert space H. Then $\forall p > 0$, $\exists$ an inverse system $\{E_m, p_n^m \mid m \geqq p\}$ with p_n^m onto, E_m open in R^m, and $E \approx$* Inv Lim E_m, *with the standard topology on* Inv Lim E_m. *Also the system $\{E_m\}$ satisfies the additional conditions*:

(a) *$\exists$ connected $m+1$ manifolds with boundary, E_{m+1}^+ and*

$$E_{m+1}^- \ni E_{m+1} = E_{m+1}^+ \cup E_{m+1}^- \quad \textit{and} \quad E_m = E_{m+1}^+ \cap E_{m+1}^-.$$

(b) $E \simeq$ Dir Lim E_m.

We also have the converse.

THEOREM B. *Given an inverse system $\{E_m, p_n^m \mid m \geqq p\}$, with p_n^m onto and E_m open in R^m, satisfying the following conditions*:

(a) *E_m splits E_{m+1} into sets E_{m+1}^+ and*

$$E_{m+1}^- \ni E_{m+1} = E_{m+1}^+ \cup E_{m+1}^- \quad \textit{and} \quad E_m = E_{m+1}^+ \cap E_{m+1}^-.$$

(b) Inv Lim E_m *is open in LR^m.*

Then Inv Lim E_m *can be embedded as an open subset of $H \ni E_m$ is embedded as a smooth submanifold. Also* Dir Lim $E_m \simeq$ Inv Lim E_m.

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