A NONSOLVABLE GROUP OF EXPONENT 5

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THEOREM 1. *There exists a group* $\mathcal{G}$ *of exponent* 5 *which is locally nilpotent, but not nilpotent. In particular,* $\mathcal{G}$ *is not solvable.*

Thus there exist varieties which are nonsolvable, but locally finite and locally solvable.

To prove Theorem 1, we first show that a certain ring is not nilpotent. Let R be the free associative ring of characteristic 5 generated by noncommuting indeterminates $x_1, x_2, \cdots$, and let L be the Lie ring in R generated by $x_1, x_2, \cdots$ where addition in L is the same as in R and Lie multiplication is commutation $[x, y]=xy-yx$ in R. An element of L will be called a Lie element.

THEOREM 2. *If we impose on* R *the following identical relations for Lie elements* x *and* y:

(i) $$x^3 = 0$$

and

(ii) $$x^2y - 3xyx + 3yx^2 = 0$$

then the resulting ring is not nilpotent.

REMARK. Higgins in [3] showed that (i) and (ii) holds in the endomorphism ring of the additive group of a Lie ring satisfying the third Engel condition.

Also worth mentioning is the following result which is equivalent to Theorem 2 as shown in Walkup [8].

THEOREM 3. *There exists a Lie ring of characteristic* 5 *which satisfies the third Engel condition and which is not nilpotent.*

G. Higman [4] and A. I. Kostrikin [5] showed that a Lie ring of characteristic 5 satisfying the fourth Engel condition is locally nilpotent, and in view of Theorem 3, this is the best one can say.

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