ON STAR-INVARIANT SUBSPACES

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Let H^2 denote the usual Hardy class of functions holomorphic in the unit disk. Let M denote a closed, invariant subspace of H^2. The theory of such subspaces is well known; every such M has the form $M=\phi H^2$, where $\phi \in H^2$ is an inner function, $\phi = Bs\Delta$, with

$$B(z) = \prod_{\nu=1}^{\infty} \frac{\bar{a}_\nu}{|a_\nu|} \frac{z - a_\nu}{1 - \bar{a}_\nu z}, \qquad s(z) = \exp\left\{-\int_0^{2\pi} \frac{e^{i\theta}+z}{e^{i\theta}-z}\, d\sigma_s(\theta)\right\},$$

$$\Delta(z) = \exp\left\{-\sum_{\nu=1}^{\infty} r_\nu \frac{e^{i\theta_\nu}+z}{e^{i\theta_\nu}-z}\right\}$$

where $\{a_\nu\}$ is a Blaschke sequence ($\bar{a}_\nu/|a_\nu| \equiv 1$ if $a_\nu = 0$), σ_s is a finite, positive, continuous, singular measure, and $r_\nu \geqq 0$, $\sum r_\nu < \infty$.

Less is known about the "star-invariant" subspaces $M^\perp = H^2 \ominus M$. In this announcement, we outline some results we have obtained recently concerning the subspace $M^\perp$. Full details and proofs will appear elsewhere.

1. **A unitary operator.** In our first theorem, we represent $M^\perp$ unitarily as the sum of the spaces $L^2(d\sigma_B)$, $L^2(d\sigma_s)$ and $L^2(d\sigma_\Delta)$. Here σ_B is the measure on the positive integers which assigns a mass $1-|a_k|$ to the integer k; σ_Δ is the measure on $[0, \infty]$ which is r_k times Lebesgue measure on the interval $[k-1, k]$; and σ_s is the measure associated with s above.

In the special case $\phi = B$, our unitary operator $V_B : L^2(d\sigma_B) \to (BH^2)^\perp$ is given by

$$V_B(\{c_n\})(z) = \sum_{n=1}^{\infty} c_n (1 + |a_n|)^{1/2} B_n(z)(1 - \bar{a}_n z)^{-1}(1 - |a_n|).$$

Here B_n is the partial product of B with zeros $a_1, \cdots, a_{n-1}$. The fact that V_B is unitary is a consequence of the simple and well-known fact that the functions $h_n(z) = (1-|a_n|^2)^{1/2} B_n(z)/(1-\bar{a}_n z)$ form an orthonormal basis of $(BH^2)^\perp$.

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