EXPRESSION FOR A FUNCTION IN TERMS OF ITS SPHERICAL MEANS

BY H. RHEE

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Let $f(X)$ be a continuous function in R^n. The spherical means, SM, of f is defined as follows:

$$\mathrm{SM}[f; X, \rho] = \omega_n^{-1} \int_\alpha f(X + \rho_\alpha) d\omega_\alpha,$$

where $X = (x, x_2, x_3, \cdots, x_n)$ is the center of the sphere of radius ρ. α denotes a unit vector. When $\rho = x$, we write $\mathrm{SM}[f; X, x] = \mathrm{SM}^*f$. The main purpose of this paper is to derive an expression for a function $f(X)$, $X \in R_+^n$ (the open half-space with $x > 0$), in terms of SM^*f. For $(X, t) \in Q_+$ ($|t| < x$, $-\infty < x' < \infty$, $x' = (x_2, x_3, \cdots, x_n)$, $(x, x') \in R_+^n$, n odd $\geqq 3$) we define the paraboloidal means, PM, of f as follows:

$$\mathrm{PM}[f; X, t] = \omega_{n-1}^{-1}(x + t)^{2-n} \int_b^\infty dy \int_\alpha f(y, x' + R\alpha) R^{n-3} d\omega_\alpha,$$

where $b = (x - t)/2$, $Y = (y, y')$, $R = [(x+t)(2y - x + t)]^{1/2}$.

A function $f(X)$ is said to belong to the class C_ϵ in R_+^n, if f is continuous in R_+^n and $f(X) = O(|X|^{(1-n-2\epsilon)/2})$, $0 < \epsilon < 1$, for large $|X|$. We observe that $\mathrm{PM}[f; X, t]$ exists, if $f \in C_\epsilon$. It is easily verified that if $f \in C_\epsilon$, then $\mathrm{SM}^*f \in C_\epsilon$. The well-known identity on iterated spherical means by John and Asgeiersson [3] states

$$(1) \qquad \int_\xi d\omega_\xi \int_\eta F(r\xi + s\eta) d\omega_\eta = 2\omega_{n-1} \int_{|r-s|}^{r+s} J\tau d\tau \int_\zeta F(\tau\zeta) d\omega_\zeta,$$

where $J = [((r+s)^2 - \tau^2)(\tau^2 - (r-s)^2)]^{(n-3)/2}(2rs)^{2-n}$.

THEOREM. *Let $f \in C_\epsilon$ in R_+^n (n odd $\geqq 3$), and let $W(X, t) = (x+t)^{n-2}\mathrm{PM}[\mathrm{SM}^*f; X, t]$. Then the following identity holds for $(X, t) \in Q_+$,*

$$(2) \quad t\mathrm{SM}[f; X, t] = M_1 D D_0^{n-3} W(X, t) + M_2 \sum_{i=1}^{(n-3)/2} a_i D_1^i t^{i+1} \mathrm{SM}[f; X, t],$$

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