A SIMPLE NONLINEAR DYNAMIC STABILITY PROBLEM[1]

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In this paper we consider a relatively simple problem to illustrate a method for treating nonlinear dynamic stability problems. This method is believed to be the first to treat arbitrary, though small, initial perturbations. Specifically, we consider

$$(1) \qquad -u_t + u_{xx} = \lambda f(u) \quad \text{for } 0 < x < \pi, \qquad t > 0$$

subject to the boundary conditions

$$(2) \qquad u = 0 \quad \text{at} \quad x = 0, \pi$$

and the initial condition

$$(3) \qquad u(x, 0; \epsilon) = h(x; \epsilon) \sim \epsilon h(x).$$

Here the symbol $\sim$ denotes asymptotic equivalence, and ϵ is a small parameter to be defined below. We assume that $f(u)$ can be expanded in a Taylor series in u about $u=0$ with $f(0)=0$ $f'(0)<0$, $f''(0)=0$, $f'''(0)>0$, and that $h(0)=h(\pi)=0$.

This problem is as a mathematical model for the temperature distribution in a bar with a nonlinear heat source of magnitude $-\lambda f(u)$, on the boundary of which the temperature is prescribed to be zero. We wish to study the stability of the equilibrium temperature distribution $u_0 \equiv 0$. To do so, we must determine whether it can sustain itself against perturbations (to which all physical systems are subjected). That is, we must see whether all perturbations decay to zero, or whether some perturbations grow (perhaps into new stationary (time independent) solutions of (1)–(2)). Therefore we study the time development of the solution with initial condition $h(x)$ representing the perturbation. The stability of solutions $u_0 \not\equiv 0$, and satisfying nonhomogeneous boundary conditions, is treated in similar fashion by considering the problem for $\bar{u} = u - u_0$.

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[1] Another version [1] of the method presented in this paper, was delivered at the S.I.A.M. Conference on Qualitative Theory of Differential and Integral Equations held at Madison, Wisconsin in August 1968.