GROWTH RATE OF GAUSSIAN PROCESSES WITH STATIONARY INCREMENTS

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Communicated by David Blackwell, November 3, 1969

1. Statement of results. Let $(Y,\ _t,\ t \geqq 0)$ be a real, separable Gaussian process with stationary increments, mean 0, and $Y_0 = 0$. Let $2Q(t)$ be the variance of Y_t and define

$$X_t = Y_t/(2Q(t))^{1/2}.$$

THEOREM 1. *Suppose there exists a nonnegative function* $v(t)$ *such that*

$$(*) \qquad \lim_{t \to \infty} \frac{Q(s+t) - Q(s)}{v(s+t) - v(s)} = 1 \quad \textit{uniformly in } s$$

and there exist positive constants s_0, β_1, β_3 *with* $1 \leqq \beta_3 \leqq (\beta_1/2+1)$ *such that*

(i) *is monotone nondecreasing,*
(ii) $v(\lambda s) \geqq \lambda^{\beta_1} v(s) > 0, \qquad s \geqq s_0,\ \lambda \geqq 1,$
(iii) $v(\lambda s) \leqq \lambda^{\beta_3} v(s), \qquad s \geqq s_0,\ \lambda \geqq 1$

and suppose that there exists $\beta_2 > 0$ *such that*

(iv) $Q(t) = O(t^{\beta_2}), \qquad t \downarrow 0.$

Then

$$\limsup_{t \to \infty} (X_t - (2 \log \log t)^{1/2}) = 0 \quad a.s.$$

In fact somewhat more is true.

THEOREM 2. *Under the assumptions of Theorem* 1,

$$\lim_{T \to \infty} \left(\sup_{t \leqq T} X_t - (2 \log \log T)^{1/2} \right) = 0 \quad a.s.$$

An important class of examples is obtained by taking $Y_t = \int_0^t Y'_s ds$ where (Y'_s) is a real stationary Gaussian process with mean 0 and continuous sample functions. If $q(|t-s|)$ is the covariance of the (Y_t)-process and $R(t) = \int_0^t (q(s) ds$ then $Q(t) = \int_0^t R(s) ds$. If $v(t)$ is a differentiable function satisfying conditions (i), (ii) and (iii) of The-

AMS Subject Classifications. Primary 6030, 6069.

Key Words and Phrases. Gaussian processes, stationary increments, law of the iterated logarithm, limiting distribution of the maximum.