HARMONIC ANALYSIS ON SEMISIMPLE LIE GROUPS

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1. **Introduction.** Let G be a locally compact group which we assume to be separable and unimodular. Let dx denote the Haar measure on G. If π is a unitary representation of G on a Hilbert space $\mathfrak{H}$ and $f \in L_1(G)$, we write

$$\pi(f) = \int_G f(x)\pi(x)dx.$$

Then $\pi(f)$ is a bounded operator on $\mathfrak{H}$ and

$$\pi(f * g) = \pi(f)\pi(g) \quad (f, g \in L_1(G)),$$

where $f * g$ denotes the convolution of f and g.

Let A be a bounded linear operator on $\mathfrak{H}$. We say that A is of the trace class if the series

$$\sum_i |(\psi_i, A\psi_i)|$$

converges for every orthonormal base $\{\psi_i\}_{i \in J}$ of $\mathfrak{H}$. Moreover if this is so, we define

$$\operatorname{tr} A = \sum_i (\psi_i, A\psi_i).$$

Then $\operatorname{tr} A$ is actually independent of the choice of this base.

Let V_π denote the set of all $f \in L_1(G)$ such that $\pi(f)$ is of the trace class. Then V_π is a linear subspace of $L_1(G)$. Put

$$\Theta_\pi(f) = \operatorname{tr} \pi(f) \qquad (f \in V_\pi).$$

Then Θ_π is a linear function on V_π which we may call the character of π. Of course this concept would be useful only when the space V_π is fairly large.

Let $\mathcal{E}(G)$ denote the set of all equivalence classes of irreducible unitary representations of G. It is easy to see that for any representation π, V_π and Θ_π depend only on the class ω of π. Hence we may de-

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