ON THE CLASSIFICATION OF LIE ALGEBRAS OF PRIME CHARACTERISTIC[1]

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This paper presents results on the classification of finite-dimensional simple Lie algebras of prime characteristic. Unlike the simple algebras of characteristic zero, these algebras do not necessarily possess a nondegenerate trace form—an important tool in classifying the characteristic zero algebras. Those which do possess such a form have been classified by G. B. Seligman [9] and, not unexpectedly, have been found to be analogs of the simple algebras of characteristic zero. However, further large classes of a quite different nature exist. Having no characteristic zero analogs, they are said to be of nonclassical type. Important work on the classification of such algebras has been done by A. I. Kostrikin ([5] and a long series of earlier papers) and by Kostrikin and I. R. Šafarevič ([7] and [8]). In an attempt to provide a uniform classification theory, they have considered the infinite Lie algebras over the complex numbers (see [1], [2], [3]). These algebras are said to be of Cartan type and have been completely classified. By replacing the complex numbers in the construction by an algebraically closed field of prime characteristic, Kostrikin and Šafarevič have produced a unified way of describing all the known nonclassical simple restricted Lie algebras. This has been further generalized by R. L. Wilson [10] to include all the known simple algebras of nonclassical type. The fact that their construction exhausts the known simple restricted algebras has led Kostrikin and Šafarevič to conjecture that there are no more such algebras. This conjecture has been proved in a very special case [7, Theorem 1]. As has developed in their work, the classification problem splits into two parts: proving the existence of a long filtration and using the filtration in an attempt to classify the algebras. By a filtration of a finite-dimensional Lie algebra L is meant a chain of subalgebras

$$L = L_{-1} \supset L_0 \supset \cdots \supset L_r \supset L_{r+1} = 0,$$

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[1] This research is contained in the author's doctoral dissertation written under the direction of Professor Richard E. Block at the University of Illinois, Urbana, and the University of California, Riverside.